AN INVESTIGATION ON SEVENTH GRADE STUDENTS' MATHEMATICAL AND ENVIRONMENTAL LEARNING RESIDUALS IN MODEL-ELICITING ACTIVITIES

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ABSTRACT

AN INVESTIGATION ON SEVENTH GRADE STUDENTS' MATHEMATICAL AND ENVIRONMENTAL LEARNING RESIDUALS IN MODEL-ELICITING ACTIVITIES

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The purpose of the study was to examine 7th grade students' learning residuals about mathematics and environmental issues in model-eliciting activities that were particularly designed to address environmental issues. The study was conducted with fourteen 7th grade students from a public middle school in Istanbul in the Spring semester of the 2020-2021 academic year. The participants were selected through purposive and convenience sampling. A qualitative educational case study was used as the design of the study. The data collection tools used in this study were two model-eliciting activities, a post-activity participant form, audio and video recordings of the implementation and a semi-structured interview conducted by students individually. The data were analyzed using the content analysis method. The findings of this study related to mathematical learning residuals showed that the students developed and/or used powerful models with multiple mathematical ideas. The findings of this study related to environmental learning residuals indicated that the students raised awareness for (1) understanding the local environmental situation and (2) developing action strategies for a sustainable future. In the light of these findings, it was suggested that model-eliciting activities that address environmental issues could be used for middle school students to teach mathematics, transform mathematical ideas into real-life situations, and raise awareness for environmental issues by integrating mathematics and science.

Keywords: Mathematical Modeling, Model-Eliciting Activities, Environmental Education, Waste Management

ÖZ

YEDİNCİ SINIF ÖĞRENCİLERİNİN MODELLEME PROBLEMLERİNDE MATEMATİKSEL VE ÇEVRESEL ÖĞRENME KALINTILARININ İNCELENMESİ

Baktemur, Gamze
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Araştırmanın amacı, 7. sınıf öğrencilerinin çevre sorunlarına yönelik modelleme etkinliklerindeki matematik ve çevre sorunları ile ilgili öğrenme kalıntılarını incelemektir. Çalışma, 2020-2021 eğitim-öğretim yılı bahar döneminde İstanbul'da bir devlet ortaokulunda öğrenim gören 14 yedinci sınıf öğrencisi ile gerçekleştirilmiştir. Katılımcılar amaçlı ve kolay ulaşılabilir örnekleme yoluyla seçilmiştir. Araştırmanın deseni olarak nitel bir eğitsel durum çalışması kullanılmıştır. Bu çalışmada kullanılan veri toplama araçları, iki model oluşturma etkinliği, etkinlik sonrası katılımcı formu, ses ve video kayıtları ve öğrencilerle bireysel yapılan yarı yapılandırılmış görüşmelerdir. Veriler içerik analizi ile analiz edilmiştir. Bu çalışmanın matematiksel öğrenme kalıntıları ile ilgili bulguları öğrencilerin çoklu matematiksel fikirler içeren güçlü modeller kullanabildiğini ya da geliştirebildiğini göstermiştir. Bu çalışmanın çevresel öğrenme kalıntıları ile ilgili bulguları ise öğrencilerin (1) yerel çevre sorununu anlamak ve (2) sürdürülebilir bir gelecek için harekete geçmek adına farkındalık oluşturduklarını göstermiştir. Bu bulgular ışığında, ortaokul öğrencilerine matematik öğretmek ve matematiksel fikirleri gerçek yaşam durumlarına dönüştürmek için çevre

sorunlarına yönelik modelleme etkinliklerinin kullanılabileceği ve matematiği fen bilimleri ile bütünleştirerek çevre sorunlarına yönelik farkındalık yaratması önerilmiştir.

Anahtar Kelimeler: Matematiksel Modelleme, Modelleme Problemleri, Çevre Eğitimi, Atık Yönetimi

To mum and dad

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LIST OF ABBREVIATIONS

ABBREVIATIONS

NCTM: National Council of Teachers of Mathematics

MoNe: Ministry of National Education

MMP: Models and Modeling Perspective

MEA: Model-Eliciting Activity

MXE: Model-Exploration Activity

MAA: Model-Adaptation Activity

OECD: Organization for Economic Cooperation and Development

STEM: Science, Technology, Engineering and Mathematics

E-STEM: Environmental - Science, Technology, Engineering and Mathematics

CHAPTER 1

INTRODUCTION

National Council of Teachers of Mathematics [NCTM] (2000) published Principles and Standards for School Mathematics, which is a guide to develop students' mathematical skills. Principles and Standards for School Mathematics include six principals (i.e. equity, curriculum, teaching, learning, assessment and technology), five content standards (i.e. number & operations, algebra, geometry, measurement and data analysis & probability) and five process standards (i.e. problem solving, reasoning & proof, communication, connections and representations) (NCTM, 2000). Problem-solving – one of the process standards – is defined as a tool that improves students' mathematical skills (Van de Walle et al., 2013). Problem-solving is essential for doing mathematics, and it is a part of the mathematics curriculum (NCTM, 2000). Students can solve problems in real-life with the help of problem-solving processes experienced in the classroom (NCTM, 2000). To do this, they can implement problem-solving situations to a new situation (Midgett & Eddins, 2001). In addition, students learn new mathematical knowledge through problem-solving (Midgett & Eddins, 2001). In Turkey, the mathematics curriculum aims to enable students to acquire mathematical thinking and reasoning skills in the process of problem-solving (Ministry of National Education [MoNE], 2018).

The nature of mathematics in terms of problem-solving has significantly changed over the past 50 years (Lesh & Zawojevski, 2007). It is necessary to adopt a new perspective for problem-solving which should go beyond the school curriculum because of the changes in the nature of mathematics (Lesh & Zawojevski, 2007). Mathematical modeling has been regarded as a new mathematics education approach for all grade levels over the last two decades (Erbaş et al., 2014). Mathematical modeling provides students with problem-solving opportunities based on real-world situations (Asempapa, 2015) and develops their analytic thinking skills (Erbaş et al., 2014). Although modeling is not one of the content standards of the NCTM (2000), it should be combined with other contents of the curriculum. In Turkey, the mathematics curriculum aims to help students gain mathematical competence which develops a way of mathematical thinking used to solve daily-life problems, and includes usage of representations such as models (MoNE, 2018). Thus, use of mathematical modeling in mathematics lessons is necessary.

There are different mathematical modeling perspectives in mathematics education (Kaiser & Sriraman, 2006). One of them is Models and Modeling Perspective (MMP) (Lesh & Doerr, 2003). MMP is a new approach including mathematical teaching, learning and problem-solving based on the constructivist and socio-cultural view (Lesh & Doerr, 2003). Students organize, interpret and explain the meaning of real-life problems using models constructed by them in the MMP (Erbaş et al., 2014). Moreover, students express, test and revise their own models and solutions during the problem solving-process in this perspective (Lesh & Zawojevski, 2007). In the MMP, problem solvers are model developers and information processors (Lesh & Yoon, 2007).

Model-eliciting activities (MEAs) are developed for the MMP (Doerr & Lesh, 2011). MEAs are real-world client-driven open-ended problems on which students work in groups of 3 or 4, and these activities last approximately 60-90 minutes (Doerr & Lesh, 2011; Hamilton et al., 2008; Lesh et al., 2003; Maiorca & Stohlmann, 2016). MEAs are developed to elicit students' initial understanding of a given situation (Doerr et al., 2017). In the MEAs, the aim is to develop a shareable and usable model originated from a given situation (Lesh & Lehrer, 2003; Lesh & Zawojevski, 2007). MEAs enable students to develop mathematical concepts through real-world examples (Moore et al., 2015). In addition, these activities enable students to express, test and revise their way of mathematical thinking during the process (Doerr et al., 2017; Lesh et al., 2003).

Mathematical modeling includes real-life context, and it has an interdisciplinary aspect (Lesh & Doerr, 2003). Mathematical modeling - specifically MEAs - can be integrated with other disciplines because of the interdisciplinary aspect. In the 21st century, innovation is important for the sustainable economic growth of nations (Çorlu et al., 2014), and education is essential for innovation (Organisation for Economic Cooperation and Development [OECD], 2010). Integrating interdisciplinary knowledge and performing complex problem-solving are pointed out in education for innovation (OECD, 2010). At that point, STEM education is needed for innovation. STEM represents four areas of science, technology, engineering and mathematics (White, 2014). Integrated STEM education is defined as combination of the concepts of mathematics and/or science with the concepts of engineering and technology (Sanders & Wells, 2006). STEM education forms connections between STEM disciplines and other disciplines, thus improving students' learning in STEM areas and other curriculum-related areas (Gallant, 2010). Furthermore, integrated STEM education motivates students for lessons, and their interest in STEM careers is increased with the help of this education (Gallant, 2010). Moreover, STEM-educated students are problem solvers, innovators, logical thinkers and technologically literate (Morrison, 2006). One of the STEM skills is problem-solving (West, 2012). As aforementioned, mathematical modeling incorporates real-world problem-solving processes that include other disciplines in a way that is similar to the interdisciplinary aspect of STEM education. Also, in the STEM there are some key elements called models (Hallström & Schönborn, 2019). Therefore, mathematical modeling activities were suggested to be used in STEM education for teaching mathematics from primary school to higher education (Tezer, 2019).

Environmental education, which is related to science, can be accepted as one of the STEM disciplines. Environmental education is necessary for raising students' awareness of the environment, and this awareness is raised by integrating environmental knowledge into lessons through other disciplines (Jianguo, 2004). Environmental learning is learning of a variety of environment-related contents like waste management through various tasks and experiences (Rickinson et al, 2009). The aim of environmental learning is "raising awareness, prompting conceptual or behavioral change, promoting moral understanding and developing metacognitive skills" and "to enhance students' knowledge and critical thinking about the environment and society so as to enable them to participate and take action as both local and global citizens, voters, and consumers" (Rickinson et al, 2009, p. 19-20). Environmental learning should be provided to students in both science lessons and other lessons in an interdisciplinary way to achieve these goals. Mathematics education which is seen as an extensive school subject can be beneficial for environmental education to prepare people for tomorrow (Barwell, 2018). Therefore, environmental issues should be included in mathematics lessons so that students gain awareness related to environmental problems while learning mathematics at the same time.

Mathematical modeling should be integrated into mathematics education to develop students' critical thinking and problem-solving skills regardless of which perspective is used (Erbaş et al., 2014). Besides, there are many environmental problems in today's world, and most of the students are unaware of these issues. Integrating environmental issues into mathematical modeling problems can be beneficial for students to raise their awareness – to understand or realize the problems and to take actions for a sustainable future. When related literature was reviewed, it was seen that there were many studies related to middle school students' modeling processes, solutions, mathematical models with multiple mathematical concepts, experiences or difficulties in model-eliciting activities (Aliprantis & Carmona, 2003; Chan, 2008; Dedebaş, 2017; Deniz & Kurt, 2021;

Eraslan & Kant, 2015; Hıdıroğlu & Özkan Hıdıroğlu, 2017; İnan Tutkun & Didiş Kabar, 2018; Lesh & Carmona, 2003; McClain, 2003; Mousoulides et al., 2007; Mousoulides & English, 2011). Still, I have not encountered any study particularly focusing on examining middle school students' mathematical and environmental learning residuals in model-eliciting activities. It is important to focus on mathematical learning residuals since teachers can see how students adapt the mathematical topics that they have learned to daily life situations. Apart from mathematical learning residuals, it is important to focus on learning residuals of other disciplines by integrating different disciplines with mathematics. In the present study, environmental issues were used as learning residuals of other disciplines. Furthermore, it is beneficial to associate modeling problems with environmental issues since students can realize these issues and think of what they can do individually or collaboratively about these issues by integrating them into modeling problems. Therefore, the present study was planned to investigate 7th grade students' mathematical and environmental learning residuals in model-eliciting activities that were designed to address a particular environmental issue - waste management.

1.1 Purpose and Research Questions of the Study

The purpose of this study was to examine 7th grade students' learning residuals about mathematics and environmental issues in model-eliciting activities that were designed to address a particular environmental issue - waste management. Specifically, this study was conducted to answer the following research question(s).

1. What are the learning residuals of modeling activities that address environmental issues?

   1.1. What do 7th grade students learn about mathematics when they engage in modeling activities that address an environmental issue (i.e., waste management)?

1.2. What do 7th grade students learn about environmental issues when they engage in modeling activities that address an environmental issue (i.e., waste management)?

1.2 Significance of the Study

There are some studies conducted in Turkey that examine middle school students' modeling processes, solutions, mathematical models with multiple mathematical concepts, experiences or difficulties in model-eliciting activities (e.g., Dedebaş, 2017; Deniz & Kurt, 2021; Hıdıroğlu & Özkan Hıdıroğlu, 2017; İnan Tutkun & Didiş Kabar, 2018). Particularly, one study was conducted to examine how 7th grade students intertwine mathematical modeling with waste management, which is an environmental problem (Gürbüz & Çalık, 2021). In their study, although they did not focus on students' learning residuals about environmental issues specifically, they found that the interdisciplinary modeling problem raised students' awareness and affected their thoughts related to their responsibilities towards the environment. However, related research is limited in the accessible literature since there is no study particularly focused on middle school students' learning residuals about mathematics and environmental issues. Therefore, this study is expected to contribute to the related literature by investigating 7th grade students' mathematical and environmental learning residuals in model-eliciting activities that were designed to address an environmental issue - waste management.

Moreover, it is significant for a researcher to study learning residuals of students since if learning residuals are not identified, a teacher would not understand what kind of knowledge and skills students have gained through activities. In addition, a teacher would not know what is missing, and what needs to be improved in the activity. Therefore, it is significant to know learning residuals to understand higher-level thinking of students, deepen the learning experience of students, and pursue these residuals to understand how learning experiences might develop.

Besides the contribution to the related literature, this study is significant for mathematics teachers, mathematics teacher educators and textbook writers. This study is noteworthy for mathematics teachers since it may inform teachers who want to use MEAs in their lessons. This study may also be beneficial for mathematics teacher educators since they may organize teacher training programs based on these findings so that pre-service mathematics teachers may gain awareness of the mathematical and environmental learning residuals of middle school students. Turkish mathematics curriculum aims to help students gain mathematical competence which means developing and applying mathematical thinking to solve a range of problems encountered in daily life (MoNE, 2018). Thus, the findings of this study is notable for textbook writers since they may add MEAs to mathematics textbooks so that students may gain mathematical competence.

1.3 Definitions of Important Terms

Model: Models are conceptual systems that are stated by using various representations such as oral language, symbols, graphs or metaphors in order to identify or explain other systems (Lesh & Doerr, 2003). In this study, the structure of students' ways of solutions indicated students' models in MEAs.

Mathematical Model: Mathematical models center on structural characteristics of related systems (Lesh & Harel, 2003). In this study, students developed mathematical models by transforming their ways of solutions into mathematics in MEAs.

Modeling: Modeling is the process of constructing a model of a situation (Lesh & Doerr, 2003).

Mathematical Modeling: Mathematical modeling is the process of translating a real-world situation into a mathematical model (Blum, 1993).

Models and Modeling Perspective (MMP): Models and modeling perspective is one of the modeling perspectives where students express, test and revise mathematical models during the problem-solving process (Lesh & Zawojevski, 2007).

Model-Eliciting Activities: Model-eliciting activities are problem solving activities that are designed to use the models and modeling perspective (Erbaş et al., 2014; Lesh & Yoon, 2007). In this study, there are two MEAs that integrate mathematics and science (related to an environmental issue of waste and trash).

STEM Education: STEM education is a problem-solving process that uses concepts from mathematics and science by combining engineering and proper technology (Shaughnessy, 2013).

Sustainability: Sustainability is to address today's and future's needs together, and it is related to how present decisions will affect the future (Chichilnisky, 2011). The sustainability issues in this study are waste and trash issues.

Environmental Learning: Environmental learning is related to learning of environment-related contents such as climate change through variety of experiences (Rickinson et al, 2009).

Learning Residuals: Otter (1992) defined learning outcomes as learners' knowledge that they gained at the end of learning experiences. In this study, learning residuals are what students learn at the end of the model-eliciting activities that address environmental issues, which are used as the learning outcomes of the study.

Mathematical Learning Residuals: In this study, mathematical learning residuals are the mathematical topics or contexts that students used in two MEAs.

Environmental Learning Residuals: In this study, environmental learning residuals are what students understand related to an environmental situation of waste management and what students think about the actions and precautions related to this environmental situation.

1.4 My Motivation to Conduct the Study

When I was a junior student in my undergraduate program, I took an elective course related to mathematical modeling. Therebefore, I had not known what mathematical modeling was, what kind of attributes modeling problems should have, or how I could solve these problems. At the beginning of the course, I thought that these problems were hard, and I could not use them for middle school students. However, at the end of the course, after experiencing different modeling problems, my opinions changed. I thought that mathematics teachers should integrate modeling problems into their lessons since according to my experience, these problems do not have a strict way of solution or result, they attract students' attention to the lesson, and develop students' mathematical thinking and reasoning.

However, when I was a senior student in my undergraduate program, I did not see any teachers who used modeling activities or any non-routine problems in the internship period. When we shared our internship experiences with our friends, I realized that most of the middle school students had no idea about mathematical modeling or model-eliciting activities. Thus, in my master thesis, I decided to research middle school students' experiences with modeling activities. Then, with the encouragement of my thesis supervisor, I decided to study 7th grade students' learning residuals about mathematics and environmental issues in model-eliciting activities that address environmental issues. I expect that this study may guide mathematics teachers and mathematics teacher educators. The mathematics teachers who read the findings of this study can have some ideas about the usage of model-eliciting activities or integration of mathematics and environmental issues in their lessons. The mathematics teacher educators can use the findings of this study, replicate the study with different samples or different aspects of the topic, or consider the findings in designing courses in teacher education programs.

CHAPTER 2

LITERATURE REVIEW

The related literature was examined under seven sections which were mathematical model and modeling, models and modeling perspective in mathematics education, Model-Eliciting Activities (MEAs), STEM education, mathematics education and environmental sustainability, studies related to mathematical modeling/MEAs and STEM education, and studies related to mathematical modeling/MEAs and environmental education.

2.1 Mathematical Model and Modeling

Models are conceptual systems which include relations or operations constructed to explain other systems for making sense of situations, actions, or experiences that involve mathematics (Lesh & Doerr, 2003). Models might include different kinds of representations such as oral language, written symbols, graphs, metaphors or schemas in order to identify other systems (Lesh & Harel, 2003). Models can be used to comprehend complex situations that we encounter in real-life (Erbaş et al., 2014). Modeling is the process of constructing a model of a situation, and the main purpose of the modeling is explaining a real-life situation or an event by means of models involving mathematical reasoning (Lesh & Doerr, 2003).

Lesh and Doerr (2003, p.10) stated that “A mathematical model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems.” If the real model is transformed into mathematics, then it becomes the mathematical model of the existing situation (Blum, 1993). Mathematical modeling is a process of constructing a model of a situation in related conceptual systems (Lesh & Doerr, 2003) or is the process of translating a real-world situation into a mathematical model within the problem-solving process (Blum, 1993). In other words, mathematical modeling builds a bridge between real-life and mathematics so that a situation in real life can be expressed mathematically (Blum, 1993; Lesh & Doerr, 2003).

Although general principle in mathematical modeling is forming connections between real-life and mathematics, different modeling perspectives have emerged over time depending on their subjects or theoretical backgrounds. Kaiser and Sriraman (2006) developed five perspectives on modeling in mathematics education. Below, the five perspectives are briefly explained.

1) Realistic (Applied) Modeling: The main aim of this perspective is to comprehend the model and solve real-life problems. Tasks must be complex, realistic and authentic. The theoretical background of this perspective is based on Pollak's pragmatic perspective. In addition, it puts emphasis on the usage of modeling process and development of modeling competencies.

2) Contextual Modeling: The main aim of this perspective is to solve word problems. Lesh and Doerr (2003) called this perspective "*Models and Modeling Perspective,"* and according to Kaiser and Sriraman (2006), it is called "*Model Eliciting Perspective*." The theoretical background of this perspective is based on information processes approaches. This perspective is different from educational perspective because model-eliciting activities (MEAs) and models are important in contextual modeling.

3) Educational Modeling: The main aim of this perspective is to learn mathematics with the help of mathematical modeling tasks. The theoretical background of this perspective is based on integrative perspective and scientific-humanistic approach. Tasks are less complex than the tasks in the realistic modeling. This perspective is different from realistic modeling because it examines not only mathematical modeling but also mathematical learning.

4) Socio-critical Modeling: This perspective puts emphasis on socio-cultural aspects of mathematics. In other words, it is related to the role of mathematics in society. The theoretical background of this perspective is based on emancipatory perspective. This perspective is different from other perspectives because the main goal is not to focus on mathematical competencies or understanding but to show the power of mathematics and make decisions about society.

5) Epistemological Modeling: The main aim of this perspective is to promote theory development for mathematical teaching and learning. The theoretical background of this perspective is based on the scientific-humanistic perspective of early Freudenthal. This perspective is different from other perspectives because ways of modeling or ways of mathematization are prioritized in this perspective. However, it is more important to mathematize real-world situations in other perspectives.

In the next section Models and Modeling Perspective was explained in detail since theoretical framework of this study is based on this perspective.

2.2 Models and Modeling Perspective (MMP) in Mathematics Education

One of the modeling perspectives in mathematics education is Models and Modeling Perspective (MMP). Lesh and Zawojevski (2007, p. 794) stated that the aim is "to enhance students' ability to use, extend, refine, and develop those mathematical ideas that they *do* bring to bear on the problem they are solving" in the MMP. A distinctive feature of MMP is that models should be expressed, tested and revised in the problem-solving process (Lesh & Zawojevski, 2007).

Doerr and Lesh (2011) stated:

> MMP research emphasizes that, before educators rush ahead to teach things, more clarity is needed about: (a) what it means to "understand" the things

> we want students to learn and (b) how the development of such understandings can be measured and assessed (p. 249).

During the problem-solving process in the MMP, it is expected to occur changes in how givens and goals are interpreted without being subject to a single path of solution (Doerr & Lesh, 2011). In the MMP, researchers/teachers focus on conceptual knowledge development or tool (model) development in order to give information to students related to real-life decisions (Lesh & English, 2005). People understand problems with the help of their cognitive models based on the models and modeling perspective (Erbaş et al., 2014). MMP includes multi-staged-studies: (1) students engage in model-eliciting activities in the cycles of expressing, testing and revision, (2) teachers engage in students' model development activities in the cycles of expressing, testing and revision and (3) researchers engage in students' and teachers' model development activities in the cycles of expressing, testing and revision (Doerr & Lesh, 2011).

In the MMP, students are given a problem including a real-life context, and they try to enhance a mathematical model (Lesh & Zawojevski, 2007). Thus, they can be thought as model developers (Doerr & Lesh, 2011). It means that students learn about both mathematical topics and problem-solving. The MMP enables students with average ability to improve mathematically powerful models in order to explain complex situations (Lesh & English, 2005). At the same time, the MMP develops students' creative thinking abilities (Dedebaş, 2017). Moreover, students see mathematics as a useful discipline for their daily-life.

2.2.1 Model Development Sequence in the MMP

Problem-solving process in the MMP includes a four-step modeling cycle (Lesh & Doerr, 2003) which is demonstrated below in Figure 2.1. *Description* step includes mapping from real-world to the model world; the model is used to make predictions for the problem situation in the *manipulation* step; in the *translation* step, findings are transferred to real-word, and *verification* step is related to practicability of the predictions that are made.

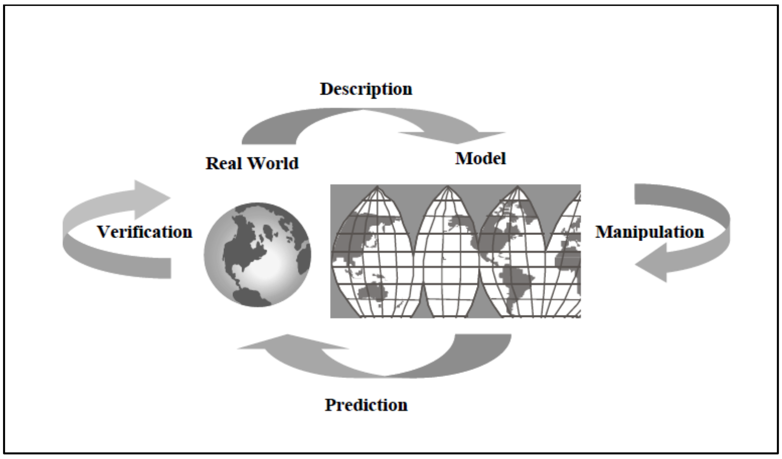


Figure 2.1 Modeling cycle (Lesh & Doerr, 2003, p.17)

Lesh et al. (2003) proposed an instructional model on the basis of the MMP. They determined three hierarchical structurally related activities which are a model-eliciting activity (MEA), a model-exploration activity (MXA) and a model-adaptation activity (MAA). In each modeling activity - MEA, MXA and MAA – the modeling cycle which is given above in Figure 2.1 occurs. In other words, modeling cycle exists in each modeling activity. The general structure of these sequences is represented below in Figure 2.2 This figure provides a model for how modeling activities can be applied to mathematics instruction and how a model development sequence can be enacted.

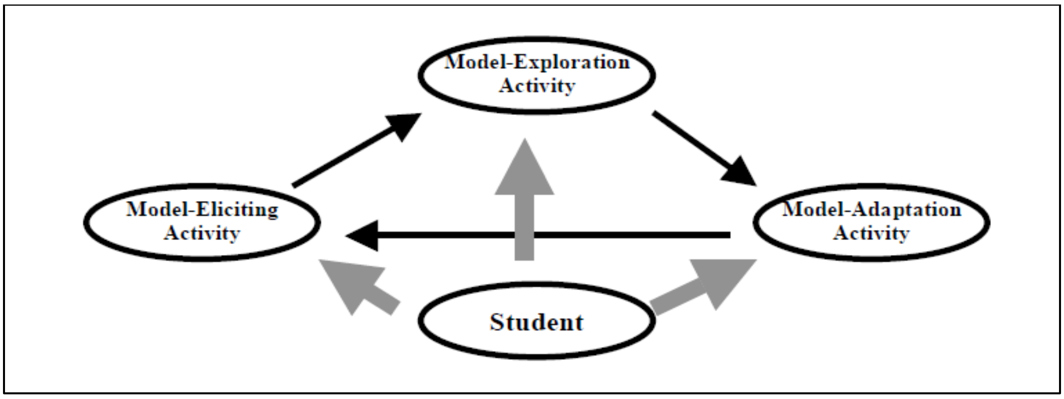


Figure 2.2 Model development sequence (Lesh & Doerr, 2003, p.17)

In the model development sequence, the first activity is a model-eliciting activity (MEA) (Lesh et al., 2003). MEAs are designed to reveal students' preliminary thinking as to a given situation (Ärlebäck & Doerr, 2015). One or two class sections are necessary to finish MEAs, and students engage in these activities in small groups (Lesh et al., 2003). The MEA is followed by a model-exploration activity (MXA). In the MXAs, the goal for students is to construct, use and experience "the language and representation system" which may include tables, graphs, animations, diagrams or algebraic representations (Ärlebäck & Doerr, 2015; Lesh et al., 2003). The MXA is followed by a model-adaptation activity (MAA). MAAs are also called model-extension activities or model-application activities (Lesh et al., 2003). In MAAs, the goal for students is to use their first model in a new situation (Ärlebäck & Doerr, 2015). The sequence continues with a new MEA at the end of the model development sequence. In brief, Doerr et al. (2017) explained these modeling activities in such a way that MEAs reveal students' initial thinking and models, MXAs enable students to think as to the models that they revealed, and MAAs provide students with the opportunity to apply their models to new situations.

Lesh et al. (2003, p. 56) stated for model development sequence that "it was designed to be used in research, as well as in assessment or instruction". It means that the sequence can be in different formats according to the purpose of use. Firstly, model-eliciting activities can be used as an independent problem-solving experience with warm-up and presentation (Lesh et al., 2003) which is represented in Figure 2.3 below. The main aim is problem solving with a model-eliciting activity. In this format, warm-up and presentation parts are not compulsory. It means that only the model-eliciting activity with model construction can be used. If these parts will be used, the process starts with the warm-up part, continues with a model-eliciting activity, and ends with students' presentations and discussions related to their models and solutions in the activity.

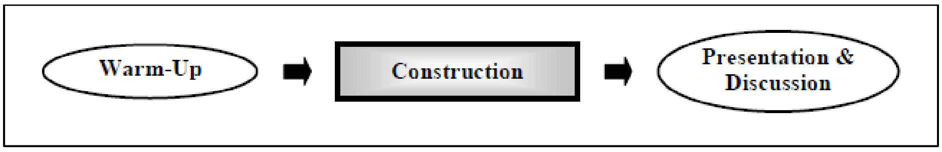


Figure 2.3 Model construction process (Lesh & Doerr, 2003, p.17)

Secondly, model-eliciting or model-adaptation activities can be used for performance assessment (Lesh et al., 2003) as in Figure 2.4 below. The main aim is assessing students before or after a traditional unit of instruction using model-eliciting or model-adaptation activities. In this format, warm-up and follow-up parts are not compulsory.

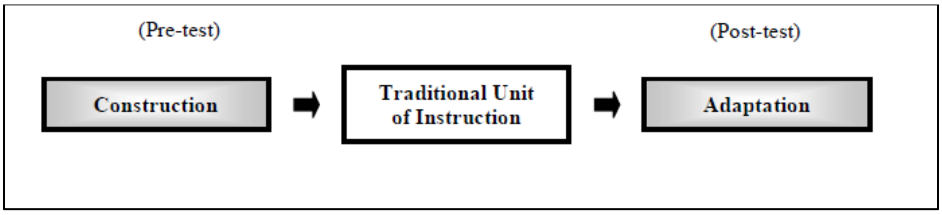


Figure 2.4 Model adaptation process (Lesh & Doerr, 2003, p.17)

Thirdly, students can experience a complete model-development sequence with MEA, MXA and MAA (Lesh et. al, 2003) which is demonstrated in Figure 2.5

below. In a complete model-development sequence that is detailed version of Figure 2.2, materials (paper-based or computer-based) and resources (books etc.) might be used during the process if necessary. The model-eliciting activity starts with a warm-up activity and continues with presentations, discussions, reflections, debriefing and follow-up activities. After the model-eliciting activity, the process continues with a model-exploration activity, again with presentations, discussions, reflections, debriefing and follow-up activities. After the model-exploration activity, process continues with a model-adaptation activity again with presentations, discussions, reflections, debriefing and follow-up activities. Lastly, the process might be finish with discussion on the structural similarities of these modeling activities.

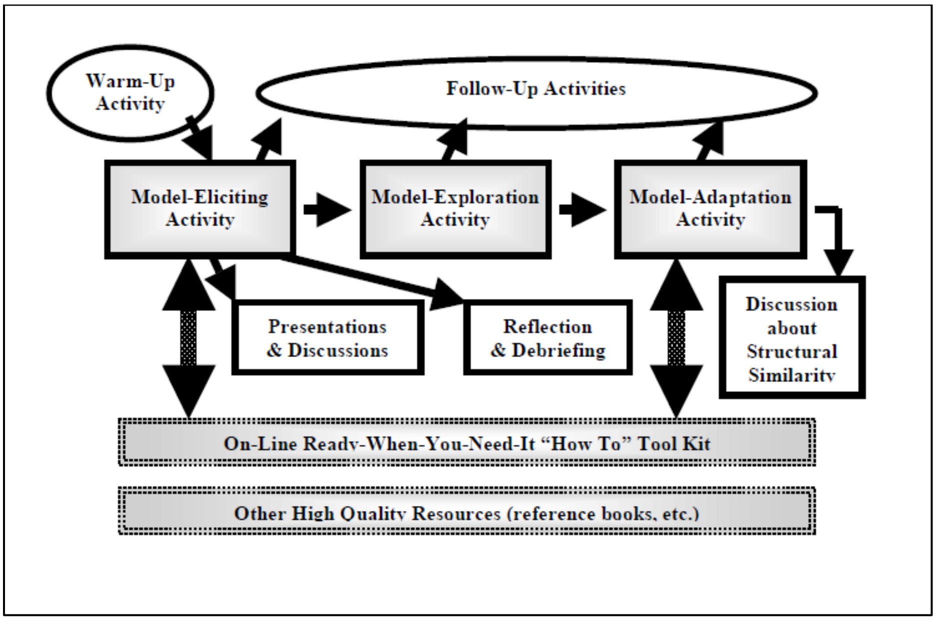


Figure 2.5 Detailed model development sequence (Lesh et al., 2003, p.57)

In the next section, the model-eliciting activities (MEAs) – principles, structural components and implementation procedure – will be explained in detail since MEAs are used as the main data collection instrument in this study.

2.3 Model-Eliciting Activities (MEAs)

Special types of problems which are called model-eliciting activities (MEAs) were designed and used in the models and modeling perspective (Erbaş et al., 2014). Model-eliciting activities are problems including a model that is revealed, and students state their solutions for the problems by testing and revising their models again and again (Lesh & Yoon, 2007). According to Lesh and Doerr (2003, p.3), model-eliciting activities "involve sharable manipulatable, modifiable, and reusable conceptual tools (e.g. models) for constructing, describing explaining, manipulating, predicting or controlling mathematically significant systems". Model-eliciting activities are purposive problems, and models that students try to develop are significant for MEAs (Lesh & Caylor, 2007). Model-eliciting activities are client-driven, complex, interdisciplinary, well-structured, non-routine problems that include real-life situations. Students work on these problems with in small groups of 3-5 students, and there can be more than one suitable solution for these problems (Chamberlin & Moon, 2005; Lesh et al., 2002; Wessels, 2014).

Model-eliciting activities are different from traditional perspective in terms of the nature of the problem, nature of mathematics, nature of mathematics teaching and nature of mathematics learning. The nature of the problem is different in MEAs because they contain a series of modeling cycles to express, test and revise an existing situation; tool development is necessary in MEAs; and the tools (models) should be reusable, modifiable and sharable (Lesh & Harel, 2003). Moreover, MEAs are more complex and realistic and include more mathematical concepts than traditional word problems (Moore et al., 2015). The nature of mathematics is different because knowledge is described by using definitions, facts or skills in the traditional perspective but it is constructed, described or explained by using mathematical models in the modeling perspective (Lesh & Doerr, 2003). The nature of mathematics teaching is different because teachers teach topics by showing facts or rules, observing student practices and correcting their misconceptions in the traditional perspective (Lesh & Doerr, 2003). However, according to Zawojewski et al. (2003, p. 353), roles of the teacher in model-eliciting activities are “to create the need for students to create significant models” and “to provide opportunities for groups to engage in multiple cycles of expressing, testing and refining their problem interpretations”. Lastly, the nature of mathematics learning is different because students learn topics by simply linking rules in the traditional perspective. On the other hand, they learn topics by integrating, differentiating or refining in the modeling perspective (Lesh & Doerr, 2003).

Usage of MEAs in mathematics lessons is important for both students and teachers. Its importance for students is that MEAs give students a chance to solve complex real-world problems by developing a mathematical model or to construct mathematical knowledge by exploration (Lesh & Doerr, 2003; Lesh & Caylor, 2007). In other words, MEAs improve students’ understanding of significant mathematical concepts (Moore et al., 2015). Its importance for teachers is that MEAs give teachers a chance to understand students’ mathematical thinking (Chamberlin & Moon, 2005).

2.3.1 Principles of Model Eliciting Activities (MEAs)

There are six principles that are important for developing model-eliciting activities (Moore, 2008). Model-eliciting activities should have planned learning goals with the help of these principles (Chamberlin & Moon, 2005). Below, the six principles are briefly explained (Chamberlin & Moon, 2005; Doerr et al., 2017; Lesh et al., 2003).

1) Reality: Ensures that scenarios of the MEAs are realistic and taken from real-life.

2) Model Construction: Ensures that the model is constructed, modified, extended, and refined, and MEAs include describing, explaining, manipulating, predicting or controlling some other systems.

3) Model Documentation: Ensures that students show their thinking process for the given situation, and write down their process technically.

4) Generalizability: Ensures that the model is re-usable, sharable and modifiable. In other words, the model should be used in other similar situations.

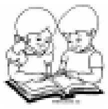
5) Self- Assessment: Ensures that students are able to evaluate their solutions by themselves.

6) Simple Prototype: Ensures that a given situation or problem is understandable and simple enough but a significant model is required.

2.3.2 Structural Components of Model-Eliciting Activities (MEAs)

An MEA consists of four parts which are the reading passage part, readiness question part, data part and problem-solving part respectively (Chamberlin & Moon, 2005). In the first part – reading passage part – students are given short attractive newspaper article(s) related to the problem statement. In the second part – readiness question part – students try to answer five or six simple questions related to the reading passage. These questions are prepared to understand students' ability to comprehend the reading passage. In the third part – data part – students are given data which can be in any form such as tables, graphs or charts. The data section is related to the readiness question part and used in the problem-solving part. In the last part – problem-solving part – students are given an MEA which is a complex non-routine problem-solving task, and they try to solve this problem for an imaginary client. An example of an MEA – Summer Reading – taken from the Model-Eliciting Activities (MEAs) Library website of University of Nevada, College of Education is given below, and each part is shown in Figure 2.6, 2.7, 2.8 and 2.9 respectively.

Summer Reading MEA



Summer Excitement Strikes the Library

Morgantown- While a long, hot summer may be ahead, the Morgantown Public Library is offering a chance for its younger patrons to stay cool. The annual summer reading program, this year titled "Reading is Radical" will officially start at noon on June 1st in the Beatrice Reading Room. Mayor Carol Hathaway will kickoff the program by reading a book to local elementary school students.

The library is celebrating the 25th year of the summer reading program. This program, which was started by three schoolteachers in the 1970s, has blossomed. Students of all ages participate annually. Over the years, several people from the community have taken part in it.

Students participating in the contest choose from an approved collection of books. The approved books have been classified by grade level, difficulty of the book, length, and genre. Students may read any of the books, regardless of their current grade level. All Morgantown students may sign up to participate in the program throughout the summer. Each student will receive a special library card to use when they sign out books for the program.

Each school has teamed up with the library to award prizes. In honor of the program's 25th year, the Morgantown Middle School Parent Teacher Organization will be awarding a five hundred dollar college scholarship to the overall winner. Numerous other prizes such as T-shirts, meals from local restaurants, computer programs, and books are available for each grade level winner.

The contest begins on June 1st and ends August 12th to give the organizers time to tabulate the points. Typically, tabulating the point totals and selecting the winners has taken a long time, so winners usually are not announced until early September. This has caused participation in the program to drop significantly in the last four years. Margaret Scott, the program director, mentioned that this year they would try to announce the winners much earlier.

Ready to go: The books are all shelved at the Morgantown Public Library. Participating students can choose from over 250 books for this year's summer reading program.

Figure 2.6 Reading passage part

As can be seen from the above figure, there is a one-page article related to Summer Reading MEA. The aim of this part is to attract students' attention to the context of the MEA.

Summer Reading Program Readiness Questions

Read the article and the tables to answer the following questions.

1. When is the program?
2. Why do the local students participate in the program?
3. What is special about the program this year?
4. Should a student receive the same number of points for The Tell-Tale Heart and Roll of Thunder, Hear My Cry? Why or why not?
5. If a sixth grader and a ninth grader both read A Tale of Two Cities, should they both earn the same number of points? Why or why not?
6. If a student reads Jurassic Park and Much Ado About Nothing, should the student get the same number of points for each? Why or why not?
7. Drew read The Tell-Tale Heart and Roll of Thunder, Hear My Cry. Should he receive the same number of points for each book? Why or why not?
8. If a sixth grader and a ninth grader both read A Tale of Two Cities, should they both earn the same number of points? Why or why not?
9. If Shelly reads Jurassic Park and Much Ado About Nothing, should she get the same number of points for each?
10. Mark read Home Run Hero and The Scarlet Letter. Should Mark receive the same number of points for both books?

Figure 2.7 Readiness questions part

In the readiness questions part, students are expected to answer short comprehension, inference or interpretation questions related to reading passage. In Figure 2.7, there are ten short questions based on the article related to summer reading program.

EXAMPLES OF APPROVED BOOKS

TITLE	AUTHOR	READING LEVEL (BY GRADE)	PAGES
Sarah, Plain and Tall	Patricia MacLachlan	4	58
Are You There God? It's Me Margaret.	Judy Blume	4	149
The Sign of the Beaver	Elizabeth George Spear	4	135
Awesome Athletes	Multiple Authors	5	288
Star Wars Jedi Apprentice: Death of Hope	Jude Watson	5	152
Encyclopedia Brown and the Case of Pablo's Nose	Donald J. Sobol	5	80
Get Real (Sweet Valley Jr. High, No.1)	Francine Pascal, Jamie Suzanne	6	144
Roll of Thunder, Hear My Cry	Mildred Taylor	6	276
The Tell-Tale Heart	Edgar Allan Poe	6	64
Talking Bout Friends	Multiple Authors	6	90
Harry Potter and the Goblet of Fire	J. K. Rowling	7	734
Little Women	Louisa Mae Alcott	7	388
The Scarlet Letter	Nathaniel Hawthorne	7	202
Home Run Hero: Sammy Sosa	Bill Gutman	7	144
Left Behind The Kids: Discovering New Believers	J. Jenkins, T. Lahaye	7	146
Aftershock (Sweet Valley High)	Kate Williams, Francine Pascal	8	208
Jurassic Park	Michael Crichton	8	400
Keeping the Moon	Sarah Dessen	8	228
In My Hands: Memories of a Holocaust Rescuer	Irene Gut Opdyke	8	248
A Tale of Two Cities	Charles Dickens	9	384
Lord of the Flies	William Golding	9	184

Figure 2.8 Data part

From the Figure 2.8, the data part includes data related to titles, authors, grade levels and pages of approved books for the Summer Reading MEA. That part is used in the next part – problem-solving part.

Summer Reading Program

Information: The Morgantown Public Library and Morgantown Middle School are teaming up to provide some of the prizes for the "Reading is Radical" summer reading contest. Participating Morgantown Middle School students in grades 6-9 will read books and prepare written reports about each book to collect points and win prizes. The winner of each grade level will be the student who has earned the most reading points. The overall winner will be the student that earned the most points. A collection of approved books has already been selected. See the previous page for a sample of this collection.

Students who enroll in the program often read between ten and twenty books over the summer. The contest committee is trying to figure out a fair way to assign points to each student. Margaret Scott, the program director, said, "Whatever procedure is used, we want to take into account: (a) the number of books, (b) the variety of the books, (c) the difficulty of the books, (d) the lengths of the books, and (e) the quality of the written reports.

Note: The students are given grades of A+, A, A-, B+, B, B-, C+, C, C-, D, or F for the quality of their written reports.

Your Mission . . .

Create a system for assigning points based on the requirements listed above. The system should be one that will allow the organizers to quickly and accurately assign and tabulate the points for each student that participates. Next, write a letter to Margaret Scott explaining how your system works. Ms. Scott hopes to find a system that will replace the current one. Please be clear and complete in your explanation.

Figure 2.9 Problem-solving part

Last part – problem-solving part – of the Summer Reading MEA includes information related to problem and students' assignment to solve it as seen in Figure 2.9. In the next sub-section, implementation procedure of model-eliciting activities (MEAs) whose structural components were given above will be explained in detail.

2.3.3 Implementation Procedure of Model-Eliciting Activities (MEAs)

Implementation procedure of MEAs consists of three parts: warm-up, modeling process and follow-up (Lesh et al., 2003). In the warm-up part, a reading passage (i.e. one-page newspaper article) and readiness questions which include five or six questions are implemented in order to attract students' attention and prepare them for the MEA. The time for warm-up part is approximately 10-15 minutes (Maiorca & Stohlman, 2016). Reading passage section can be carried out as an in-class or out-of-class activity, and students answer readiness questions in the class (Coxbill et al., 2013).

In the modeling part, students work with the modeling problem collaboratively by expressing, testing and revising their models (Coxbill et al., 2013). According to Zawojewski et al. (2003), groups should include three or four students, be determined by the teacher, and be selected heterogeneously. One copy of the MEA should be given to each student or each group, and then sufficient time - approximately 5 minutes - should be given to read the MEA (Zawojewski et al., 2003). The teacher should make sure that every student understands the problem situation. Then, groups work with the MEA during two or three class periods - approximately 60-90 minutes (Coxbill et al., 2013; Doerr & Lesh, 2011; Lesh et al., 2003). Teachers should listen and observe students to understand their mathematical thinking during the modeling part (Zawojewski et al., 2003). In addition, students are expected to record their work before the modeling part is completed (Coxbill et al., 2013), and they generally write two-page letters for an imaginary client (Lesh et al., 2003).

In the follow-up part, groups share their models in the class (Dedebaş, 2017), compare and discuss their models or solutions with other groups' solutions, make revisions, and express the modeling process or mathematical topics that they used (Maiorca & Stohlman, 2016). After the discussion, reflection or follow-up activities can be carried out. Reflection activities include short questions related to group or individual work so that students express/evaluate their feelings, attitudes or behaviors (Lesh et al., 2003). Furthermore, teachers prepare follow-up activities including a number of textbook problems so that students can see the connections between the model-eliciting activity and traditional activities (Lesh et al., 2003).

2.4 STEM Education

STEM is the acronym of four fields which are Science, Technology, Engineering and Mathematics (Fitzallen, 2015; Marrero et al., 2014). There is no consensus as to the definition of STEM education, and its definition may change based on different perspectives (Breiner et al., 2012; Martín-Páez et al., 2019; Zhou, 2010). To illustrate, according to the educational perspective, modern view of STEM education incorporates the fields of science, technology, engineering and mathematics under a single unit (Breiner et al., 2012). STEM education is the teaching/learning of two or more STEM fields or teaching/learning of one STEM field with one or more disciplines apart from STEM fields (Sanders, 2009). STEM can be defined as "pursuit of innovation" (Watson & Watson, 2013, p.1). STEM education involves a problem-solving process which benefits from concepts or methods used in science and mathematics by combining them with proper technology and engineering (Shaughnessy, 2013). According to another definition made by President's Council of Advisors on Science and Technology [PCAST] (2010), STEM is a learning environment in which students study with real-life contexts through discovery.

STEM education involves teaching and learning experiences in the four fields and contains both formal and informal activities for students from pre-school to post-doctorate (Gonzalez & Kuenzi, 2012). STEM teaching and learning include realistic contexts to meet human needs innovatively (Merrill, 2009). STEM learning is the combination of several contents to solve interdisciplinary real-life problems, and STEM teaching includes experiences like problem-solving or logical reasoning that students engage in to acquire STEM learning (Martín-Páez et al., 2019). In addition, Martín-Páez et al. (2019) stated that students enhance their STEM proficiency with these experiences of problem-solving or logical reasoning.

The concept of integration in STEM education is significant since science, technology, engineering and mathematics are taught as integrated disciplines (Breiner et al., 2012). Bryan et al. (2015) defined integrated STEM education in such a way that concepts in science and/or mathematics are taught and learnt with the integration of technology and engineering. STEM integration can be in different forms such as integrating two (i.e. engineering and math) or three disciplines, or integrating four disciplines by overlapping (Bybee, 2013). According to Smith and Karr-Kidwell (2000), the aim of integrated STEM education is integrating disciplines so that learners learn topics in a meaningful way and form connections between the topics.

Recently, the scope of the STEM is extending (Kaya & Elster, 2019), and thus new STEM areas are emerging. For instance, E-STEM that suggests an integration of environment into other STEM areas is one of the emerging versions of STEM approach (Helvacı & Helvacı, 2019). E-STEM approach is studied “by using the environment as physical context, a conceptual topic, or both, to stimulate knowledge of natural systems, while developing problem solvers equipped to tackle ecological challenges.” (Gupta et al., 2018, p. 229). E-STEM tries to solve environmental issues with STEM – in an interdisciplinary way (Kaya & Elster, 2019). In other words, in the E-STEM activities, students realize environmental problems while engaging in problem solving, raise their awareness and take actions related to these problems (NAAEE, 2013).

There are several benefits of STEM education. For instance, STEM education may inspire students to choose careers related to STEM such as chemical engineering, aerospace or architecture (Egli, 2012). It improves students’ use of technology and necessary 21st century skills of communication, problem-solving or self-management to help them become better decision-makers (Bybee, 2010). Students can solve the problems that they encounter in their lives by the virtue of STEM education (Yıldırım, 2017). It affects students' motivation in lessons positively and makes lessons more attractive (Niess, 2005). STEM areas need creativity (Marrero et al., 2014) and therefore, STEM education improves students' creativity. STEM education is not only important for students but also for countries. Countries need to be powerful in the fields of technology, economy and science (Şahin, 2019). Mathematical modeling activities are found to support STEM education in terms of addressing different disciplines and present realistic problem situations (English, 2017). Therefore, studies related to mathematical modeling and STEM are explained in detail in the following pages.

2.5 Mathematics Education and Environmental Sustainability

There are several definitions of sustainability, and there is no universal definition that is accepted (Hamilton & Pfaff, 2014). Sustainability is to continue welfare for a long time (Heinberg & Lerch, 2010; Kuhlman & Farrington, 2010). According to Chichilnisky (2011), sustainability is to address today's and future's needs together or it is related to how present decisions will affect the future.

Mathematics allows integration to maintain sustainability (Petocz & Reid, 2003) and incorporating sustainability into the teaching process is significant (Hamilton & Pfaff, 2014). Five learning objectives may be included in the curricula of most of the courses to teach sustainability (Hamilton et al., 2010). These objectives are listed below (Hamilton & Pfaff, 2014):

(1) Teach in a context. Include sustainability-oriented content and introduce "global realities" (p. 7)
(2) Include real-life place-based examples (p. 8)
(3) Emphasize "designing the future". Teach the tools of complexity, systems thinking and design thinking (p. 13)

(4) Explicitly recognize the ethical and affective (moods, feelings, attitudes etc.) aspects of the issues that are raised (p. 14)
(5) Teach specific skills that empower students to become the catalysts and leaders of change (p. 14)

To give an example of these objectives from mathematics education based on the first objective and second objective, mathematics teachers can select problems from real-life environmental issues, or based on the fourth objective, teachers may use data tables, for instance data showing the amount of sea ice in statistics lessons to increase awareness of students with regard to climate change (Hamilton & Pfaff, 2014).

Mathematics education is necessary for environmental sustainability since mathematics is needed to understand environmental problems such as pollution or climate change (Coles et al., 2013). Furthermore, citizens can attend debates on future problems or changes by the virtue of mathematics education (Barwell, 2018). Statistical literacy and mathematical modeling enable students to understand and interpret environmental problems by studying with real data and various representations (Barwell, 2018). To illustrate, climate change, which is one of today's serious problems, is described by using statistical concepts such as means or its future effects can be predicted with the help of mathematical modeling (Barwell, 2013). Moreover, Barwell (2013) stated that mathematical literacy or statistical literacy can be used for communication of climate change in such a way that data or graphs about global temperature changes are interpreted with statistical literacy.

Mathematics contributes to sustainability development in such a way that mathematics is a human activity, and knowing this promotes sustainable development (UNESCO, 2017). Generalizations and abstractions are powerful mathematical tools to make future predictions related to the needs of governments on environmental subjects (UNESCO, 2017). Mathematics enables us to make decisions on specific topics; number, operation or measuring systems and symbols address the needs related to sustainability (UNESCO, 2017). In addition, mathematical modeling may be used to reduce waste, maximize profit or predict energy efficiency (UNESCO, 2017) or may be used to examine the sustainability of biological populations (Petocz & Reid, 2003). In short, as stated by UNESCO (2017, p. 39) "mathematics is a tool for sustainable development". Therefore, environmental learning, which means learning of a variety of contents related to environment like ecosystems, waste management or climate change (Rickinson et al., 2009), should be integrated into mathematics lessons which we aimed to accomplish, in this study, by integrating one of these issues – waste management - into model-eliciting activities.

2.6 Studies Related to Mathematical Modeling/Model-Eliciting Activities and STEM Education

There are several studies related to mathematical modeling/model-eliciting activities and STEM education (Baker & Galanti, 2017; Baker et al., 2019; Güder & Gürbüz, 2018; Stohlmann et al., 2013; Suh & Han, 2019). In some of these studies, the researchers focused on the statement, "MEAs as a tool for STEM education" (Baker & Galanti, 2017; Baker et al., 2019; Güder & Gürbüz, 2018). To begin with, Baker and Galanti (2017) conducted a design-based implementation research to examine how their design decisions provided an opportunity for K-6 mathematics teachers to think of model-eliciting activities as a vehicle for STEM education. The study was conducted with four classroom teachers, three mathematics coaches, one elementary mathematics interventionist, one middle grade special educator and the division math supervisor. Daily writing reflections, discussions, Teacher Efficacy and Attitudes toward STEM Survey for Elementary Teachers and MEAs (Survivor, Packing It In, Creating a Mosaic, A Day at the Zoo) were used as data collection instruments. Qualitative analysis were used to analyze the data. The results of the study indicated that the participants started to think of model-eliciting activities as a vehicle for STEM education. They realized the difference between MEAs and problem-based learning. In addition, dealing with MEAs as learners and changing actual tasks enabled participants to think comprehensively as to MEAs & STEM integration.

Another qualitative study of Baker et al. (2019) examined the effects of two mathematics specialists' positioning on MEA implementations in K-6 classrooms which benefit from STEM disciplines. A design-based implementation research was used as the method of the study. Data were collected through observations, surveys, interviews and The Box Turtle MEA. Data were analyzed qualitatively – by coding. According to the results of the study, mathematics teachers comprehended MEAs as enjoyable activities before implementation. During the implementation, mathematics specialists created a discussion environment for teachers. On the other hand, after the implementation with the help of mathematics specialists' positioning, the teachers saw MEAs as rich mathematical problems. Moreover, the results of the study showed that MEAs help students equalize their mathematics learning in significant STEM experiences.

Güder and Gürbüz (2018) examined the opinions of teachers and students on whether interdisciplinary modeling activities are vehicles for STEM education. Semi-structured interview technique was used as the design of the study. Participants of the study were two teachers (one of them was a mathematics teacher, and other one was a science teacher) and seven 7th grade students from a middle school. Data were collected through semi-structured pre-interviews, semi-structured post-interviews and three model-eliciting activities. Data were analyzed by using descriptive analyses. The results of the pre-interviews indicated that both teachers stated that mathematics and science are related to real-life and other courses. The results of the post-interviews indicated that MEAs help interdisciplinary learning, and they should be in the curriculum according to the teachers' explanations. After the implementation of the MEAs, interviews were also conducted with the students. Based on the results of the interviews conducted with the students, it was stated that they had not seen MEAs before, their attitude toward interdisciplinary learning changed positively, and their self-confidence and attitude to mathematics were enhanced with the help of MEAs.

Other researchers examined how STEM with mathematical modeling affects pre-service teachers' competencies/mathematical knowledge (Stohlmann et al., 2013; Suh & Han, 2019). For instance, Suh and Han (2019) conducted a mixed methods research with a convergent parallel design to examine the effects of STEM project with mathematical modeling on university students' proficiency. Forty-two university students attended the study. Data sources were an 18-item survey as to students' perception on mathematical modeling, semi-structured interviews, worksheets and daily reflection sheets. Quantitative analysis was made by using t-tests, cross tab and Cronbach's alpha, and qualitative analysis was made by examining interview transcripts and identifying the themes. Based on the quantitative analysis of the study, the students noticed that mathematical modeling is a useful tool to identify problems in the present, may predict future problems as to environmental, social or economic issues, and may determine possible solutions to fulfill the future generations' needs at the end of the STEM project. Based on the qualitative analysis of the study, the students followed a modeling process different from what researchers had expected. They followed an alternative process which was different from Blum's (2011) circular process. Students also understood that modeling steps were not independent. In the last session after the presentations of students' works, they reexamined the modeling steps. Moreover, the students realized that STEM tasks with mathematical modeling were related to real-life situations, and they could make interdisciplinary connections.

Stohlmann et al. (2013) conducted a qualitative study to examine twenty-six pre-service elementary teachers' mathematical content knowledge in a STEM-modeling activity. Selected pre-service teachers completed a mathematics method course. The Lesh Translation Model (Lesh et al., 1987), which is related to five representations which are concrete, realistic, symbolic, language and pictorial, was used as a measure of content knowledge of pre-service teachers. The Bigfoot MEA was used as the STEM-modeling activity. Furthermore, audio recordings of the groups' work, the groups' written works and the researcher's field notes were used as data collection instruments. Data were analyzed by coding the pre-service teachers' content knowledge based on the representations of Lesh Translation Model and then examining the translations between representations. The results of the study showed that pre-service teachers developed their subject matter content knowledge on linear functions with the help of the Bigfoot MEA. All groups of pre-service teachers showed conceptual understanding by means of translations within and between symbolic, realistic, language, and concrete representations. On the other hand, three of the seven groups used pictorial representations.

2.7 Studies Related to Mathematical Modeling/Model-Eliciting Activities and Environmental Education

There are some studies related to mathematical modeling/model-eliciting activities and environmental issues (Gürbüz & Çalık, 2021; Mousoulides & English 2011; Mousoulides et al., 2010). Mousoulides et al. (2010) conducted a longitudinal study to explore twenty-two 11 year-old students' processes and model development through an environmental modeling problem called The Water Shortage which is related to the water shortage in Cyprus. Data were collected through audio and video tapes of the students' responses to the modeling activity, the researchers' field notes, student worksheets and Google Earth spreadsheet files. Miles and Huberman's (1994) interpretative technique was used as the data analysis method. The results of the study revealed that the students solved the environmental modeling problem by developing different models such as graphical or algebraic. In another study conducted by Mousoulides and English (2011), they studied with six groups of twenty 12 year-old high achiever students - who were a part of a study on children's mathematical modeling and engineering thinking - to examine their models and develop their mathematics and science learning while solving an engineering model-eliciting activity called Natural Gas. The data collection tools were audio tapes of the students' works, video tapes of the students' responses during class discussions, student worksheets and reports, and the researchers' field notes. The results of the study indicated that four groups developed suitable models to solve the problem. In addition, the models of two groups were more rational since they showed regard to the effects of renewable energy sources on natural gas consumption.

Gürbüz and Çalık (2021) conducted a case study with six 7th grade students to examine how students intertwine mathematical modeling with an environmental problem of waste management. Data were collected through video tapes of students' dialogues. Data were analyzed by identifying themes and categories. Results of the study showed that students learned about environmental issues that were aimed in the interdisciplinary modeling problem. In other words, students intertwine mathematics and environmental issues/science education. In addition, the interdisciplinary modeling problem raised students' awareness and affected their thoughts related to their responsibilities towards the environment based on the results of the study.

To sum up, when related literature was examined, it was seen that there are several studies related to mathematical modeling/model-eliciting activities and STEM education (Baker & Galanti, 2017; Baker et al., 2019; Güder & Gürbüz, 2018; Stohlmann et al., 2013; Suh & Han, 2019). Based on the results of these studies, some of the researchers found that the teachers saw MEAs/modeling activities as a vehicle for STEM education (Baker & Galanti, 2017; Baker et al., 2019; Güder & Gürbüz, 2018). Based on the results of other studies, it was seen that STEM with mathematical modeling activities affects pre-service teachers' competencies/mathematical knowledge positively (Stohlmann et al., 2013; Suh & Han, 2019). Furthermore, there are some studies related to mathematical modeling/model-eliciting activities and environmental issues (Gürbüz & Çalık, 2021; Mousoulides & English 2011; Mousoulides et al., 2010). Based on the results of these studies, it was seen that the researchers generally focused on mathematical models of students in modeling activities that included environmental issues. Even though there are some studies related to mathematical modeling with STEM education, and mathematical modeling with environmental issues, there is a gap in the literature since there is no study conducted to examine particularly middle school students' mathematics-related and environmental issues-related learning residuals. Therefore, the aim of this study was to examine 7th grade students' learning residuals about mathematics and environmental issues in model-eliciting activities that address an environmental issue – waste management.

CHAPTER 3

METHODOLOGY

The methodology section was discussed in detail in eight parts which are design of the study, participants, data collection tools, data collection procedure, data analysis, researcher's role, trustworthiness of the study and limitations of the study.

3.1 Design of the Study

The design of the study was educational case study, which is one of qualitative research types. In a case study, researchers focus on a case that can be a single individual or a group of individuals like a class, event or process, and study with this case (Fraenkel et al., 2012). The aim of the case studies is gaining in depth understanding of a specific case with a detailed study (Creswell, 2002; Fraenkel et al., 2012).

In this study, the researcher tried to gather in depth understanding as to 7th grade students' learning residuals related to mathematics and environmental issues in model-eliciting activities that address an environmental issue – waste management. Therefore, the case of the study was a group of 7th grade students who had experienced mathematical modeling.

3.2 Participants

The participants of the study were 14 seventh grade students studying in a public middle school in Sancaktepe, İstanbul. The school is located on the Anatolian side of İstanbul. There were 36 classrooms, a special-education classroom, a science laboratory, a chemistry laboratory, a painting class, a music class, 49 teachers and 1214 students who were at the school during the second semester of 2020 – 2021 academic year. The school was not an eco-school, students did not participate in any environmental activities, and only the standards of the middle school mathematics and science programs of MoNE were followed. Class sizes were ranged between 40 – 44 students. There were three project classes from 5th, 6th and 7th grades on the school. Project classes were determined based on students' elementary school scores when they started the 5th grade. Project classes consisted of approximately 30 students. Project classes take English-based education in their first year of middle school. It means that they take more English lessons than other 5th grade classes but other lessons are the same. Achievement levels of the students were variable in the middle school. It means that the students were at high, moderate or low achievement levels. The socio-economic status of the students was also variable. It means that there were students from families whose socio-economic status was high, middle or low. The researcher has been a mathematics teacher in this school from the beginning of the 2020-2021 academic year.

Seven of the students participating in this study were girls, and seven of them were boys. Their ages ranged between 12 and 13. Thirteen of the students were selected from the project class, and one of the students was selected from another 7th grade class which was not a project class. One student was selected from another 7th grade class since the student was very successful and interested in mathematics based on the researcher's observations. Also, the student was willing to participate in extracurricular activities. Five groups were formed with these 14 students. Four groups were consisted of three students, and one group was consisted of two students. Eight of the students were very successful in mathematics lessons, and six of them were average students based on the researcher's observations. On the other hand, all of the students were interested in mathematics lessons, always attended the lessons and had basic mathematical skills. Socioeconomic status of the participants was middle class. The participants' average mathematics scores in 6th grade and in the first semester of the 7th grade are given in Table 3.1.

Table 3.1 Mathematics scores of the participants

Name	**6th grade math scores (%)**	**1st semester of 7th grade math scores (%)**
Student 1	94	100
Student 2	98	100
Student 3	87	82
Student 4	92	87
Student 5	92	85
Student 6	100	98
Student 7	77	77
Student 8	94	96
Student 9	91	100
Student 10	83	85
Student 11	81	90
Student 12	88	78
Student 13	93	88
Student 14	97	85

In the study, purposive and convenience sampling methods were used to select the participants. Qualitative samples are in tendency to be purposive since the special context of a case is the main focus of researchers (Miles et al., 2018). In this study, the researcher focused on a group of 7th grade students who had mathematical modeling experience. In the convenience sampling, participants are selected based on the suitability of the researcher and study (Fraenkel et al., 2012). Convenience sampling was used since the researcher was the mathematics teacher of two classes that the students were selected from. Thus, the researcher knew the students' mathematics achievements, abilities and personality traits. Another reason why these students were selected for the study was that their mathematics achievement was average or above. In model-eliciting activities, students should be able to use the required mathematical knowledge to develop a model and solve the problem. In the study, selected students were capable of using basic mathematical concepts. Furthermore, the participants were selected among students who did not have any problems with the internet and device access since the study was carried out online.

3.3 Data Collection Tools

Data collection tools that were used in the study were model-eliciting activities, a post-activity participant form, audio and video recordings and semi-structured interviews.

The first model-eliciting activity was *Trash Trouble* which is given in Appendix A. This activity was adapted from the *Trash Trouble MEA* in the website of the Model-Eliciting Activities (MEAs) Library of University of Nevada, College of Education. While adapting the Trash Trouble, the context of the problem – the trash issue in America – was transformed into the trash issue in İstanbul, therefore it constituted a local issue for the students. MEA had a news article and readiness questions related to the article based on the environmental issue included in the problem. The MEA was adapted to create a procedure or formula for predicting the amount of trash that Istanbul will produce in 2025. Related mathematics contents in the first MEA were pattern and generalization, arithmetic average and ratio and proportion. Related environmental issue in the first MEA was reducing the amount of trash and recycling.

The second model-eliciting activity was *Minimum Waste, Maximum Pencil Box* which is given in Appendix B. This activity was adapted from the *Coffee Cup MEA* in the website of the Model-Eliciting Activities (MEAs) Library of University of Nevada, College of Education. While adapting the Coffee Cup, the context of the problem – coffee cup – was transformed into metal pencil box since cylinder and net of the cylinder (circle and rectangle) were familiar mathematical concepts for the students. The MEA had a news article and readiness questions related to the article based on environmental issue included in the problem. The MEA was adapted to develop a model to minimize the amount of waste materials when making the bottoms and sidewalls of a metal pencil box for an engineering and architectural company. Related mathematics contents in the second MEA were ratio and proportion, right circular cylinder, rectangle, circle and square. Related environmental issue in the second MEA was reducing the amount of waste.

For the content-related evidence of validity of the MEAs, expert opinions were obtained while adapting the problems from the thesis supervisor who was interested in mathematical modeling and from a mathematics education professor. To do this, the content and format of the MEAs were checked by the thesis supervisor based on the six design principles of model-eliciting activities. Then, the MEAs were revised based on the supervisor's feedbacks.

The post-activity participant form which includes four short questions is given in Appendix C. The form was used as a reflection activity with the aim of understanding the students' learning related to mathematics and environmental issues at the end of each MEA. In addition, this form was used so that the students could express/evaluate their learning.

Observations were made by the researcher during the implementation stage of the MEAs. The researcher had the role of participant-as-observer during observations since the researcher participated in the situation during the process (Fraenkel et al., 2012). Since the study was conducted online, the researcher joined each Breakout Room on Zoom (that is, visited each group one by one) and observed the students' work. Moreover, the researcher took notes related to the groups' works by visiting each room. Since it is difficult to observe students on an online platform, audio and video recordings were necessary for the researcher. Therefore, to support the observations and use them in the data analysis process, the model-eliciting activity sessions were recorded on Zoom when it was possible. It means that since it was not possible to record Breakout Rooms when the researcher was not there, the researcher made recordings while visiting each room (each group). In addition, whole group discussions and presentations were recorded.

Semi-structured interviews were conducted to gain in-depth understanding of the 7th grade students' learning related to mathematics and environmental issues in model-eliciting activities. To do this, the interviews were conducted with one student from each of the five groups. Those students were selected according to their ability to express themselves and to be more active in the modeling process. The main questions asked during the interviews are given below:

1. How did you and your groupmates solve the problem?
2. What did you learn about mathematics when you engaged in the problem?
3. What did you learn about the environmental issues when you engaged in the problem?

In addition to the main questions, elaborating and probing questions were asked to students based on their answers to understand their thinking/solution process deeply.

For triangulation of the semi-structured interview; audio and video recordings were used, and during the interviews, notes were taken by the researcher related to the students' answers. Moreover, expert opinions were taken from the thesis supervisor to check whether questions were suitable.

3.4 Data Collection Procedure

Necessary permissions were taken before the data collection process. First of all, the ethical committee approval, which is given in Appendix D, was taken from the Middle East Technical University Human Subjects Ethics Committee. Secondly, permission taken from the Ministry of National Education (MoNE) as shown in Appendix E. Then, since the participants were under the age of 18, permission was taken from their parents by sending them a form which included the purpose of the study and request for including their children in the study. Parent consent form, which are given in Appendix F, were filled out online because of distance education.

Work schedule for the implementation process of model-eliciting activities is given in Table 3.2 below. The procedure is also explained in detail below.

Table 3.2 The schedule for the implementation procedure of MEAs

Date	**Duration**	**Related part of the implementation procedure**
19.04.2021	20 minutes	Online meeting
25.04.2021	as homework	Warm-up (reading passage and readiness questions) for MEA-1
26.04.2021	5 minutes	Answering the readiness questions for MEA-1
26.04.2021	85 minutes	Modeling process of MEA-1
27.04.2021	45 minutes	Follow-up (group presentations, discussion and revision) for MEA-1
02.05.2021	as homework	Warm-up (reading passage and readiness questions) for MEA-2
03.05.2021	5 minutes	Answering the readiness questions for MEA-2
03.05.2021	85 minutes	Modeling process of MEA-2
04.05.2021	45 minutes	Follow-up (group presentations, discussion and revision) for MEA-2

Because of Covid-19 pandemic, the students had to take distance education, and lesson duration decreased by 10 minutes. So, there was not enough time in the school time to conduct the study. Therefore, the students attended online classes at a different time from the school time for the implementation procedure. The time of these online classes was determined with students based on their and the researcher’s convenience.

A twenty-minute online meeting was held with the students to inform them as to the study before the implementation procedure. The information included the brief purpose, approximate duration and procedure of the study. In addition, the researcher explained that student names or IDs would not be used anywhere, the results of the study would not affect their mathematics grade or teachers’ attitude, and if they do not want to continue, they can leave the study at any time. Actually, the students' names should not be asked to ensure the confidentiality but in this study, after the implementation procedure of model-eliciting activities, the students filled out post-activity participant forms as homework. Thus, their names were asked to examine what each student learnt about mathematics and environmental issues.

Implementation process of model-eliciting activities was carried out in three sessions: warm-up, modeling process and follow-up. For the warm-up part, reading passages – news articles related to the environmental issue in each MEA – were assigned to students as homework because of time limitation. The students read the passage and answered the questions by themselves before the in-class section of the modeling process part. Therefore, this part was done as an out-of-class activity individually. Before starting the modeling process part, the students discussed the news articles briefly by answering the readiness questions as an in-class activity. This part lasted approximately 5 minutes. In addition, the students shared their answers with the researcher before the modeling process part by writing down their answers and taking and sending the photos of their answers.

Modeling process part was carried out online – using the Zoom platform. The reason why this part was carried out online was Covid-19 pandemic. 7th grade students in the Istanbul had online education during the data collection process, and there was no opportunity to make the study face to face. After answering the readiness questions, the problems were demonstrated to the students by the researcher in each MEA. Before starting the group work, the MEAs were read, and necessary explanations were made by the researcher to make sure that each student understood the problem. This section lasted approximately 5 minutes for each MEA. Then, the researcher separated the students into groups, and the problems were shared with each group. The students were divided into five groups. Four groups were consisted of three students, and other group was consisted of two students. The groups were selected before the implementation process by the researcher to make them heterogeneous. Group members were the same in the both problems. Breakout Rooms in the Zoom platform were used for group work.

Before the group work of the second MEA - Minimum Waste, Maximum Pencil Box – the researcher gave necessary information related to the right circular cylinder briefly since the second MEA was related to the right circular cylinder. To do this, the students were informed about what right circular cylinder is and what kind of geometric shapes a cylinder is comprised of. In addition, the area of rectangle, area of square and area of circle were remined to students. This section was carried out outside group work time. The duration of this session was approximately 15 minutes.

After demonstrating the problems and grouping the students, they started to work on modeling problems with their groupmates. Students worked with their groupmates by turning on either their microphones or both microphones and camera. They took notes digitally and on paper during the problem-solving process. Furthermore, some of the groups took screenshots because they made some drawings and computations on the screen. During the group work, the researcher visited the Breakout Rooms frequently, took notes and asked questions to students as to their solutions. In addition, the researcher recorded Zoom sessions as much as possible while visiting the groups. This session lasted approximately 80 minutes for each MEA without any break.

Follow-up part for each MEA was carried out on the day following the modeling process part because the students were tired after 80 minutes of online lesson without any break. Thus, follow-up part was carried out the next day to make presentations and discussion more meaningful and productive. The groups shared their models and solutions for the problems during the follow-up part as a whole-group activity. To do this, one or two students from each group explained their solutions and models. In addition, the students from other groups compared and discussed the solutions of the presenting group by asking questions. After group presentations, the researcher asked students which mathematical topics they used and what they learnt about environmental issues. This session was also carried out as a whole-class discussion. Then, the researcher grouped the students again and gave time to groups to revise their solutions. This session lasted approximately 45 minutes for each MEA. There was no follow-up activity after the presentations and revision because of time limitation.

After the implementation of each model-eliciting activity, reflection activity was carried out as an out-of-class activity. The students filled out the post-activity participant form as homework because of time limitation. Then, they shared their post-activity participant forms by taking and sending photos or filling out the forms online with the researcher.

After the implementation of model-eliciting activities, semi-structured interviews were carried out with one student from each group to understand groups' solutions better. In total, interviews were carried out with 5 students online – using the Zoom platform. The duration of interviews was approximately 15-20 minutes.

3.5 Data Analysis

In this study, content analysis, which is a qualitative data analysis method, was used. To do this, firstly the audio and video recordings of the groups' works were transcribed. Secondly, the audio and video recordings of semi-structured interviews were transcribed. Then, the written works of the students, the audio and video recordings of the groups' works and the audio and video recordings of semi-structured interviews were coded based on research questions. Hence, data-driven coding frame was used in the content analysis. Coding is a way of discovering the meaning of data (Saldana, 2011). The data in this study were coded through two cycles (1) holistic coding and (2) descriptive coding (Miles et al., 2018). In the first cycle, the data were examined holistically, and general codes were identified rather than examining the data in detail. In the second cycle, general codes identified in the first cycle were grouped into a smaller number of categories. During the coding process, the written work of the students, the audio and video recordings of the groups' works and the audio and video recordings of semi-structured interviews were coded all together based on research questions.

3.6 The Researcher's Role

Researcher bias is a potential threat to validity in qualitative studies and may affect the results of the study (Johnson, 1997). Creswell (2009) stated that researcher in qualitative studies is an inquirer and is involved in the study with participants. Thus, researcher should explain his/her experiences with students and the relationship between students and researcher to decrease the bias (Creswell, 2009).

As the researcher, I have been a mathematics teacher in this school from the beginning of the 2020-2021 academic year. Thus, I was their mathematics teacher. In addition, I was the main class teacher of 13 students selected as participants from the project class. At the beginning of the study, I informed the students as to the brief purpose, approximate duration and procedure of the study. I explained them that their names, IDs or audio and video recordings would not be used anywhere, and the results of the study would not affect their mathematics grade or teachers' attitudes. I selected 14 students as participants based on their mathematics' grades and my observations during the lessons. That is because students should have basic mathematical skills to solve modeling problems. I also take into consideration students' interest in mathematics. I did not guide or give direction to the students about how they could solve the problems during the modeling problem-solving process in order not to affect the results of the study. During the semi-structured interviews, I just tried to understand in detail how they solved the problems and did not make any comments. During the data collection process, I was respectful and nonjudgmental. I reported the findings of the study fully and honestly.

3.7 Trustworthiness of the Study

Lincoln and Guba (1985) identified four themes for validity and reliability issues of a qualitative research as mentioned below. Since the design of the study was qualitative, these four issues were briefly discussed.

Credibility is internal validity in a qualitative research and is related to whether the study measures what is actually intended (Shenton, 2004). In the study, triangulation and prolonged engagement were used to ensure credibility. Triangulation was used since there are multiple data sources: model-eliciting activities, researcher's observation notes, audio and video recordings, post-activity participant form, students' field notes, students' written works or drawings, and semi-structured interviews. Prolonged engagement was used because the researcher has been the mathematics teacher of the students since the beginning of 2020-2021 fall semester. It means that the researcher stayed in the setting for a long time. Thus, the students were relaxed during the study. In addition, two coders – the researcher and thesis supervisor – coded the students' work in the model-eliciting activities and the students' answers in semi-structured interviews. Thus, consistency between two coders were examined. Thick description, which means knowing the context of the study in detail to help other researchers use the researcher's findings, was used to ensure *transferability*. The number of participants, their characteristics, context of the study, instrumentation issues and data collection process were explained in detail in the parts above so that other researchers may transfer the results of the study. *Dependability* is reliability in qualitative research and is related to finding similar results by replicating the study with the same context, participants and method (Shenton, 2004). Detailed explanation was given related to the context, participants and method to ensure dependability. In addition, detailed explanations related to the study (e.g. participants, design, instrumentation or procedure) and the researcher's role were provided in previous sections to ensure *confirmability*. Furthermore, the students' work in the model-eliciting activities and the students' answers in semi-structured interviews were coded by both the researcher and thesis supervisor.

3.8 Limitations of the Study

There were three limitations of the study. The first limitation was related to the number of participants. The study was conducted with only 14 students. Nevertheless, the results of the study may be useful for other researchers, mathematics teacher educators and/or mathematics teachers if they study with a group of middle school students whose traits are similar to those of the participants in this study.

Second limitation of the study was related to distance education. The study was conducted online education because of Covid-19 pandemic. Thus, the students had to do group work in an online environment and did not work face to face. Interaction between group members was not as good as in the classroom setting. The researcher tried to eliminate this limitation as much as possible by asking students to open their cameras and microphones during the process. In addition, the researcher had difficulty observing the students while they were working in Breakout Rooms. Zoom sessions were recorded as much as possible to overcome this difficulty. For this purpose, both audio and video recordings and note-taking were used.

Third limitation of the study was related to time. Two lessons of 40 minutes (in total 80 minutes) were given for each MEA in Zoom without any break. Since the study was conducted outside the school time, it was hard to organize a suitable timetable for all students. Therefore, students had to work for 80 minutes for the modeling process part of each MEA. Therefore, there was not enough time for any follow-up activity. This limitation may be eliminated by conducting the study face-to-face but it was not possible during the 2020-2021 academic year. In addition, the researcher visited Breakout Rooms frequently, asked questions related to their work and tried to encourage them to work in order to reduce the disadvantage of not having a break, motivate the students and maintain their attention.

CHAPTER 4

FINDINGS

The findings section was discussed in three parts which are students' solutions to environmental based MEAs, mathematical learning residuals and environmental learning residuals.

4.1 Students' Solutions to Environmental Based MEAs

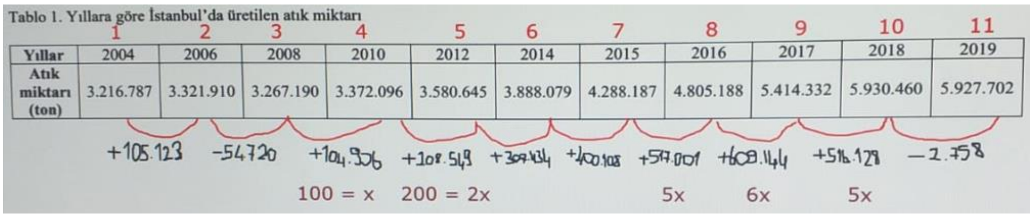
4.1.1 MEA 1 – Trash Trouble

The first MEA was Trash Trouble which consisted of two questions. The first question of the problem was related to creating a procedure or formula for predicting the amount of trash that will be produced in Istanbul in 2025. In the first question, the students were expected to find how much trash would be produced. The second question of the problem was, "What should be the amount of trash in order to produce 650.000 MWh of electricity from landfill gas in 2025?" In the second question, the students were expected to find the amount of trash to produce 650.000 MWh of electricity from landfill gas.

Mathematical learning residuals that I - as the researcher - expected from students was to use pattern and generalization, algebra, arithmetic average and/or ratio and proportion. Also, I expected them to find the amount of increase/decrease in the amount of trash between years while solving the problem. Environmental learning residuals that I - as the researcher - expected from students was to realize the issue of trash/waste and to seek a solution for this issue.

4.1.1.1 Solutions of the Group 1 in the Trash Trouble

Amount of trash in 2025. When written works, presentations and semi-structured interviews of the students from the first group were examined, it was seen that at the beginning of the study, the students thought that they could solve the first question by using ratio and proportion or pattern and generalization. They started with the numbered years and wrote the amount of increase/decrease between the amount of trash which is shown in Figure 4.1 below.


Tablo 1. Yıllara göre İstanbul'da üretilen atık miktarı

	1	2	3	4	5	6	7	8	9	10	11
Yıllar	2004	2006	2008	2010	2012	2014	2015	2016	2017	2018	2019
Atık miktarı (ton)	3.216.787	3.321.910	3.267.190	3.372.096	3.580.645	3.888.079	4.288.187	4.805.188	5.414.332	5.930.460	5.927.702

Figure 4.1 Group 1's work to find the amount of increase/decrease in the amount of trash in the Trash Trouble MEA

As can be seen from the Figure 4.1, the years were numbered from 1 to 11 on the table by the students in the 1st group. Then, they found the amount of increase/decrease in the amount of trash in the given 11 years. For example, they found an increase of 400.108 from 2014 to 2015.

Then, they tried to generalize the amount of increase by using algebra. To do this, they accepted an increase of approximately 100.000 as x. Thus, they wrote 2x for an increase of approximately 200.000, and so on. After that, they tried to find a pattern between the amounts of increase. They realized that there was an increase of 5x from 2015 to 2016, 6x from 2016 to 2017, and 5x from 2017 to 2018. Hence, they determined the pattern of 5x, 6x and 5x which is shown in Figure 4.1 above. However, they did not take into account the decrease of 2.758 from 2018 to 2019.

Lastly, they added an approximate amount of trash until reaching 2025 by using the pattern they had determined as can be seen in Figure 4.2.

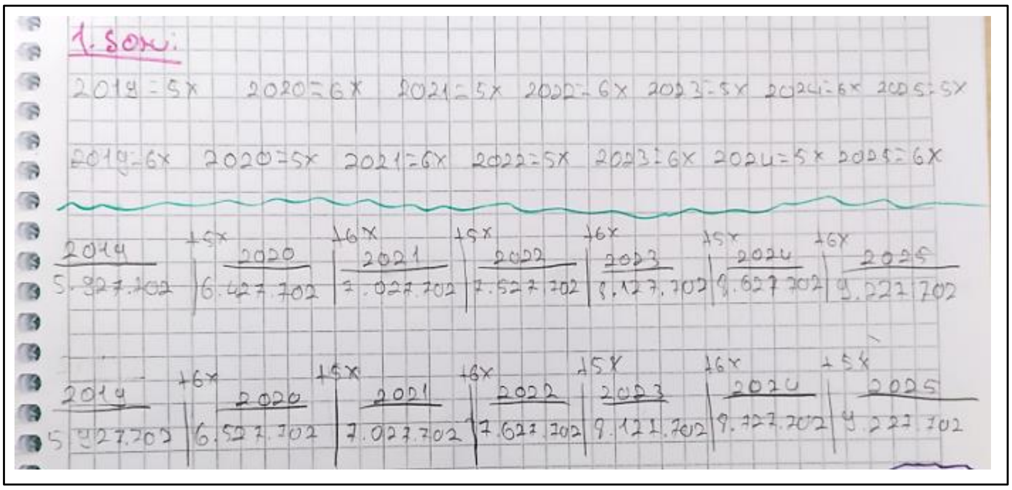


Figure 4.2 Group 1's work to add an approximate amount of trash in the Trash Trouble MEA

As understood from the figure above, the students in the 1st group added the amount of trash in two different ways but they reached the same answer of 9.227.702. One of the students used the pattern of 5x, 6x, 5x, and another student used the pattern of 6x, 5x, 6x. This was important from the modeling perspective since they tried to construct a model by generating an algebraic pattern.

Below is the conversation related to the students' solution for the first question of Trash Trouble MEA between Student 2 and the researcher in the semi-structured interview.

> *Student 2:* First of all, 11 years are given in the table. We numbered these years from 1 to 11. Then, we found the amount of increase between the amount of trash. Then, we noticed that there was an increase of approximately 100.000 from 2008 to 2010. We also noticed that there was an increase of approximately 200.000 from 2010 to 2012, an increase of approximately 300.000 from 2012 to 2014, an increase of approximately 400.000 from 2014 to 2015, an increase of approximately 500.000 from

2015 to 2016, and an increase of approximately 600.000 from 2016 to 2017. Then, we said x for an increase of approximately 100.000.

Researcher: It means that you said x for an increase of approximately 100.000, 2x for an increase of approximately 200.000, and so on.

Student 2: Yes. Then, we saw that this continued as 5x, 6x, 5x. Thus, we thought that this would continue as 5x, 6x, 5x. According to Student 1, after 2019, the amount of increase should start with 5x. According to me, the amount of increase should start with 6x. Then, we continued to examine the amounts according to both of our opinions. At the end, we found the same answer of 9.227.702.

Researcher: Okey, at this point I have two questions. Firstly, what did you do with -2.758. I mean, from 2018 to 2019, there was a decrease. What happened to this decrease?

Student 2: Since it has always increased so far, and the amount of decrease here is very small, we ignored it. We did not take it into account.

Researcher: Okey, secondly, you continued to add the amount of trash starting with 5x, and Student 1 continued to add the amount of trash starting with 6x. At the end, you found the same answer. Why?

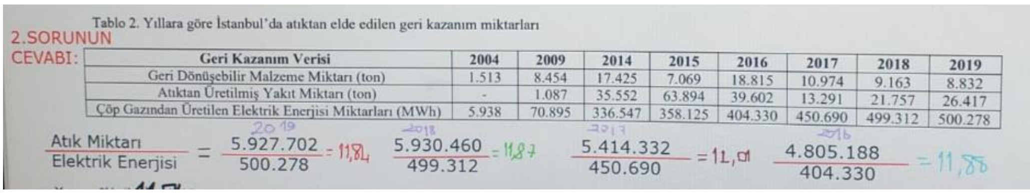
Student 2: I think, teacher, it is because as we noticed there are 3 5x and 3 6x in Student 1's answer. I also have 3 5x and 6 3x. As a result, both of us have the same answer.

Researcher: Okey, do you think we should have started from 5x or 6x? Why?

Student 2: I think, teacher we should have started from 6x since from 2017 to 2018 there was an increase of 5x. We ignored the decrease of 2.758 from 2018 to 2019. Thus, after 5x there should be 6x to keep the pattern of 5x, 6x, 5x.

This conversation also showed that the students found that there could be 9.227.702 tons of trash in 2025 by using an algebraic pattern.

Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. When the first group's solution for the second question was examined, it was seen that they used ratio and proportion and arithmetic average. Figure 4.3 demonstrates that they tried to find a fixed ratio by dividing the amount of trash by electrical energy produced from landfill gas.


2.SORUNUN CEVABI:

Tablo 2. Yıllara göre İstanbul'da atıktan elde edilen geri kazanım miktarları

Geri Kazanım Verisi	2004	2009	2014	2015	2016	2017	2018	2019
Geri Dönüşebilir Malzeme Miktarı (ton)	1.513	8.454	17.425	7.069	18.815	10.974	9.163	8.832
Atıktan Üretilmiş Yakıt Miktarı (ton)	-	1.087	35.552	63.894	39.602	13.291	21.757	26.417
Çöp Gazından Üretilen Elektrik Enerjisi Miktarları (MWh)	5.938	70.895	336.547	358.125	404.330	450.690	499.312	500.278

$$\frac{\text{Atık Miktarı}}{\text{Elektrik Enerjisi}} = \frac{5.927.702}{500.278} = 11{,}84 \quad \frac{5.930.460}{499.312} = 11{,}87 \quad \frac{5.414.332}{450.690} = 11{,}01 \quad \frac{4.805.188}{404.330} = 11{,}88$$

Figure 4.3 Group 1's work to proportion the amount of trash by electrical energy produced from landfill gas in the Trash Trouble MEA

As shown in Figure 4.3, the students divided the amount of trash by electrical energy in 2016, 2017, 2018 and 2019 and found a ratio for each year. For instance, they divided 5.927.702 by 500.278 by using the data of 2019 and found 11.84. On the other hand, it can be seen that they did not take into consideration the first four years given in the table.

Then, they found the approximate average of these ratios as 11.86 and used this average as the fixed ratio. Figure 4.4 shows that they used the ratio of $\frac{amount\ of\ trash}{electrical\ energy}$ and found the answer as 7.709.000. This was important from the modeling perspective since they tried to construct a model with this ratio.

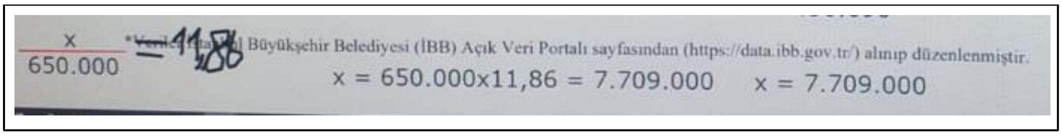


Figure 4.4 Group 1's work to find the amount of trash to produce 650.000 MWh of electricity from landfill gas in the Trash Trouble MEA

From the figure above, we can see that the students used a fixed ratio of 11.86 which corresponds to a value founded by dividing the amount of trash by electrical energy in any year. However, at that point, they made some computational mistakes while calculating the fixed ratio. Then, they used this ratio for 2025, too. To do this, they divided x (amount of trash) by 650.000 and equalized this to 11.86. Lastly, they found 7.709.000 tons of trash by using cross-multiplication.

Below is the conversation related to the students' solution for the second question of Trash Trouble MEA between Student 2 and the researcher in the semi-structured interview.

> *Student 2:* In the second question, we tried to find how much trash there should be in 2025 by dividing the amount of trash by electrical energy. According to the data of 2019, we divided the amount of waste by energy, and found 11.84. Similarly, we found 11.86 for 2018, 12.01 for 2017 and 11.88 for 2016. Then, we found the approximate average of these divisions and found 11.86.
>
> *Researcher:* What does 11.86 mean?
>
> *Student 2:* This is the approximate value we get when we divide the amount of trash by electrical energy in any year.
>
> *Researcher:* Okey, go on.
>
> *Student 2:* Then, we said x for the amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. We divided x by 650.000, and this

> division is equal to 11.86. We made cross multiplication to find x. It means that we multiplied 650.000 by 11.86 and found the answer as 7.709.000.

This conversation indicates that the students found the answer of 7.709.000 by using a fixed ratio and arithmetic average.

Researcher's account of Group 1's model of the Trash Trouble. On the basis of the answer of the 1st group, their solution was based on algebra, pattern and generalization, and ratio and proportion mathematically. The reason for the use of algebra was that they said x for approximately 100.000 increase. The reason for the use of pattern and generalization was that they determined a pattern that continued as 5x, 6x, 5x, 6x. The reason for the use of ratio and proportion was that they wanted to find a fixed ratio by dividing the amount of trash by electrical energy in the second question. The model that I deduced as the researcher from the solution of the students in the first group for the first question is given in the algebraic expression below.

$$+5x,\ +6x,\ +5x,\ +6x,\ \ldots$$

As a result, it is possible to see an algebraic pattern of +5x, +6x. It means that an amount of 5x and 6x could be added, respectively. The students reached this conclusion with mathematical thinking and algebraic reasoning.

The model that I inferred as the researcher from the solution of the students in the first group for the second question is given in the expression below. As can be seen from the expression below, the students reached a ratio which was equal to a fixed number of 11.86.

$$\frac{amount\ of\ trash}{amount\ of\ electrical\ energy\ produced\ from\ landfill\ gas} = 11.86$$

As a second result, if the amount of trash is proportioned by the amount of electrical energy produced from landfill gas, a fixed value is obtained. This value could be found by taking the ratio of the amount of trash to the amount of electrical energy in any year. The students reached this conclusion with mathematical thinking and proportional reasoning.

4.1.1.2 Solutions of the Group 2 in the Trash Trouble

Amount of trash in 2025. The students in the second group made a guess without making any computation while finding the amount of trash. Firstly, they examined the changes in the amount of trash. Then, they made a guess with these changes. Below is the related explanation for the amount of trash in 2025 taken from the second group's written field notes.

> *Group 2:* … If we give an amount, we think the amount of trash in 2025 will be around 7.600.000.…We did our calculations as follows:
>
> -We looked at the amount of trash between years. We observed that there was a different increase each year. We saw an increase of one million in 12 years, from 3 million in 2004 to 4 million in 2015. Since there was a ten-year difference between 2015 and 2025, this was an increase of around 2.5 million. This was our guess.

As can be deduced from the sentences above, the students did not make an exact computation. They just made a guess. They thought that from 2014 to 2015, the amount of trash increased by about one million, which is equivalent to a 12-year increase. Since it is ten years from 2015 to 2025, they thought that a ten-year difference would correspond to an increase of 2.5 million and found the answer as 7.600.000. However, they did not explain how they reached these conclusions.

Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. When works of the students in the second group were examined, it was seen that they did not do any work related to the second question.

When the students in group 4 were asked what they did about the second question during the group presentation, they said that they did not find the answer to the second question since they had not enough time. As can be understood, the students had a problem with time and did not work on the second question. Thus, the students could not conclude the second question.

Researcher's account of Group 2's model of the Trash Trouble. Based on the works of the 2nd group, the students did not reach a mathematical model and made a guess with regard to the problem in the first part. The reason might be that they worked for 85 minutes during the modeling process part. The remaining time was not enough for them to create a mathematical model.

4.1.1.3 Solutions of the Group 3 in the Trash Trouble

Amount of trash in 2025. At the beginning of the study, the students in the third group thought that they could solve the first question by using pattern and generalization. They thought that they should start by finding the amount of increase/decrease between the amount of trash. Then, they decided to use arithmetic average concept and found the average. (See Figure 4.5)

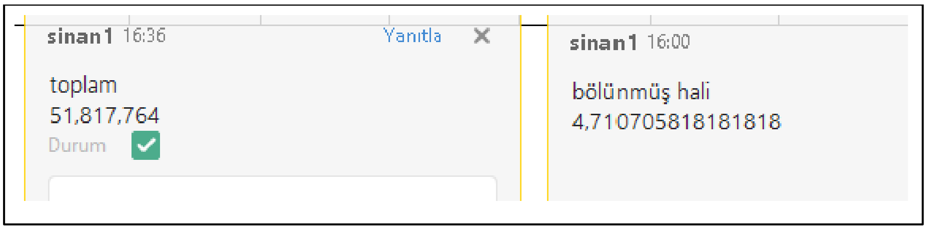


Figure 4.5 Group 3's work to find the average amount of trash in Trash Trouble MEA

As seen, the students in the 3rd group found the sum of the trash as 51.817,764. Then, they divided this sum by 11 (the number of years) and found the answer as approximately 4,71. This means that the amount of trash per year could be equal to the average amount of trash that they found. This was important from the modeling perspective since they tried to construct a model to find the average amount of trash per year.

Below is the conversation related to the students' solution for the first question of Trash Trouble MEA between Student 8 and the researcher in the semi-structured interview.

> *Student 8:* In the table, there were 11 years. We did not look at the differences between years. We found the sum of the trashes in these 11 years. We found the sum as 51.817,764. Then, we found the divided version.
>
> *Researcher:* What does divided version mean?
>
> *Student 8:* It means that we divided the sum by the total number of years which is 11. We found the result as 4,710705818181818.
>
> *Researcher:* What did you actually find by doing this process?
>
> *Student 8:* We found how much trash there could be on average per year. We actually found the answer. We thought that we could find the approximate amount of trash in 2025 by finding the average amount of trash.

This conversation between the student and the researcher also points out that the students found the amount of trash in 2025 as approximately 4,71 million by using arithmetic average.

Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. At the beginning, the students in the third group again used arithmetic average concept for the second question. To do this, they found the sum of the electrical energy produced from landfill gas for the given 8 years. Then, they divided the sum by 8 and found the answer as 386.56,263.

Below is the conversation related to students' solution for the second question of Trash Trouble MEA between Student 8 and the researcher during the group presentation.

Student 8: Teacher, actually our answer is wrong. We found an answer similar to our answer in the first question. That is to say, we added the amount of electrical energy produced from landfill gas for the given eight years which you gave in the table. Then, we divided it by 8. Our answer was 386.56,263. However, our answer is wrong.

Researcher: Why do you think so?

Student 8: Because in the question you asked us what the amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025 should be but we did not take into consideration the amount of trash. We just found the average amount of electricity.

Researcher: Okey, you can revise your solution.

Student 8: Okey, teacher. We wanted to revise, but we did not have enough time before the presentations.

This conversation highlighted that the students found the average amount of electricity as the answer. On the other hand, they ignored the amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025.

After they revised their solutions, the third group found answer as 7.476.086 tons of trash as can be seen in Figure 4.6.

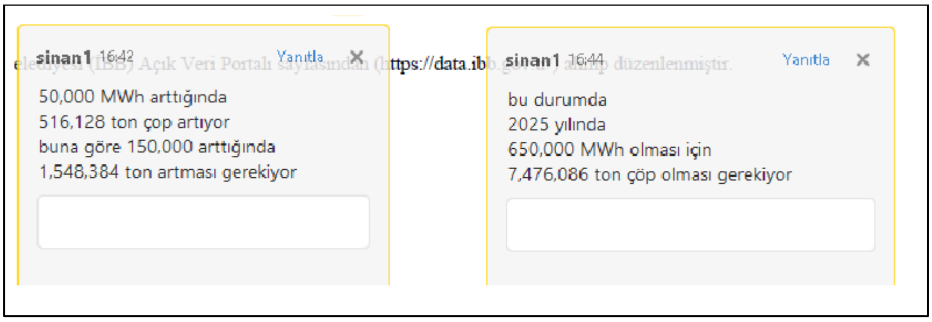


Figure 4.6 Group 3’s work after the revision in the Trash Trouble MEA

As Figure 4.6 shows, the students used ratio and proportion to solve the second question. They found that approximately 50.000 MWh of electricity increased, and the amount of trash increased by 516.128 tons from 2017 to 2018. Based on the question, electricity produced from landfill gas should be 650.000 MWh in 2025. This means that from 2019 to 2025, electricity should increase by 150.000 MWh. The students noticed that since 150.000 MWh is equal to three times 50.000 MWh, they multiplied 516.128 tons of trash by 3. In short, they reached the answer of 7.476.086 tons of trash. This was important from the modeling perspective because they tried to construct a model with ratio and proportion. However, it can be seen that the students just focused on the years 2017 to 2018 and did not examine the data of other years.

Below is the conversation related to their solution for the second question of Trash Trouble MEA after revision between Student 8 and the researcher in the semi-structured interview.

> *Student 8:* Teacher, from 2017 to 2018, the amount of electricity from landfill gas increased by 50.000 MWh. From 2017 to 2018, the amount of trash increased by 516.128 tons. In 2019, there was 500.278 MWh of electricity. From 2019 to 2025, the amount of electricity should be increased by 150.000 MWh so that electricity will be 650.000 MWh in 2025. If we look, 150.000 MWh is equal to three times 50.000 MWh. If we multiply 516.128 tons by 3, we get 7.476.086 tons of trash which is the answer.
>
> *Researcher:* Why did you multiply 516.128 by 3?
>
> *Student 8:* Since we multiply the amount of electricity, we should multiply the amount of trash to fix the ratio.

As understood from the conversation above, after group presentations, the students revised their solution and reached 7.476.086 tons trash as the result.

Researcher's account of Group 3's model of the Trash Trouble. According to the answer of the 3rd group, they solved the problem by using arithmetic average and ratio and proportion mathematically. The reason for the use of arithmetic average was that they wanted to find the amount of trash in 2025 by using the average amount of trash there could be per year. The reason for the use of ratio and proportion seemed to be that they wanted to find the amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025 by using build-up strategy (between ratio model). The model that I obtained as the researcher from the solution of the students in the third group for the first question is given in the expression below. As can be seen from the expression below, the students reached a ratio which is equal to the average amount of trash per year.

$$\frac{total\ amount\ of\ trash}{total\ number\ of\ years} = average\ amount\ of\ trash\ in\ per\ year$$

From this result, it can be inferred that if the total amount of trash is divided by the total number of years, the average amount of trash per year is obtained. The students reached this conclusion with mathematical thinking and arithmetic reasoning.

The model that I interpreted as the researcher from the solution of the students in the third group for the second question in given in the expression below.

$$\frac{\begin{matrix} the\ amount\ of\ increase\ in\ the \\ amount\ of\ electricity \\ from\ landfill\ gas \\ between\ two\ years \end{matrix}}{\begin{matrix} the\ amount\ of\ increase\ in\ the \\ amount\ of\ electricity \\ from\ landfill\ gas \\ between\ other\ two\ years \end{matrix}} = \frac{\begin{matrix} the\ amount\ of\ increase \\ in\ the\ amount\ of\ trash \\ between\ two\ years \end{matrix}}{\begin{matrix} the\ amount\ of\ increase \\ in\ the\ amount\ of\ trash \\ between\ other\ two\ years \end{matrix}}$$

This result shows that if build-up strategy (between ratio model) is used, the ratio that shows how much electricity increases from between two consecutive years to between two other consecutive years might be found. Similarly, this ratio is be equal to how much trash increased from between the same two years to between the same other two years. The students reached this conclusion with mathematical thinking and proportional reasoning.

4.1.1.4 Solutions of the Group 4 in the Trash Trouble

Amount of trash in 2025. The students in the fourth group thought that they could solve the problem by using ratio and proportion. At first, they found the difference between the amount of trash in 2019 and 2004. Then, they divided this difference by 15 as illustrated in Figure 4.7.

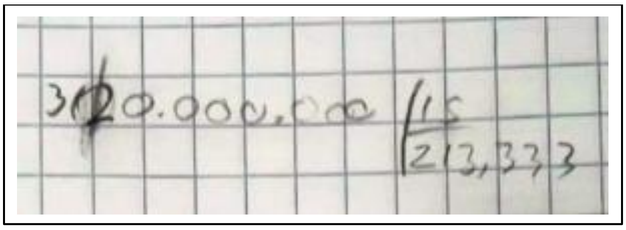


Figure 4.7 Group 4’s work to find how much more trash was produced in a year in the Trash Trouble MEA

Figure 4.7 illustrates that the students divided 3.000.000 (the approximate difference between the amount of trash in 2019 and 2004) by 15 (total number of years between 2004 and 2019) and found 213.333. 213.333 which corresponded to how much more trash was produced in a year. This was important from the modeling perspective because they tried to construct a model with unit ratio.

Then, Figure 4.8 shows that they added 213,333 tons of trash to each year, starting from 2019 until 2025.

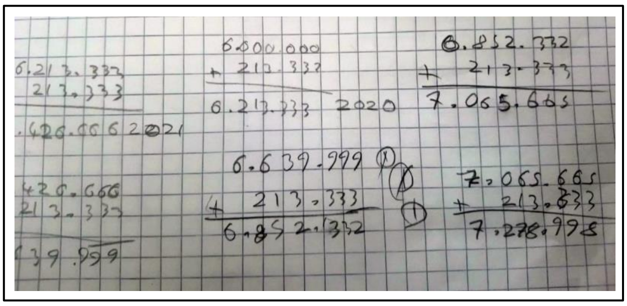


Figure 4.8 Group 4’s work to add trash produced in a year in the Trash Trouble MEA

Figure 4.8 demonstrates that the students added six times 213.333 tons of trash until 2025 in total.

Lastly, they found the answer as 7.278.998 as can be seen in Figure 4.9.

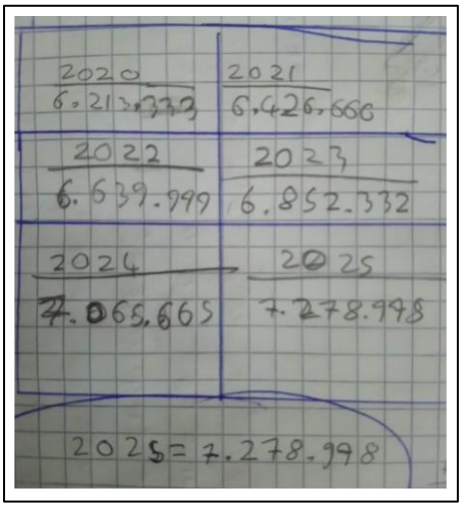


Figure 4.9 Group 4’s work to find amount of trash in 2025 in the Trash Trouble MEA

Figure 4.9 indicates that they reached the answer of 7.278.998 for 2025.

Below is the conversation related to the students’ solution for the first question of Trash Trouble MEA between Student 10 and the researcher during the group presentation.

> *Student 10:* First, we found the difference between 2019 and 2004. It means that we found how much more trash was produced. It is 3 million. 3 million tons of trash were produced in 15 years. Then, we divided 3 million by 15.
>
> *Researcher:* Why did you divide 3 million by 15?
>
> *Student 10:* Since there are 15 years in total.
>
> *Researcher:* Okey, go on.

Student 10: From here, we found the answer as 213.000.

Researcher: What does 213.000 correspond to?

Student 10: It corresponds to how much more trash is produced in a year. Then, we added one by one. In 2019, there were 6 million tons of trash. We added 6 million and 213.000 and found 6.213.333. We added other years like that. Lastly, we found that there would be 7.278.998 tons of trash in 2025.

From the conversation above, the students found 7.278.998 tons of trash at the end. They used unit ratio, found the amount of increase in each year and added that amount until reaching 2025. However, they had some calculation mistakes.

Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. Similar to their first solution, the students in the fourth group used unit ratio. They found the difference in the electricity produced from landfill gas between 2019 and 2004 as 494.000. Then, they divided 494.000 by 15 and found the answer as 698.000.

Below is the conversation related to the students' solution for the second question of Trash Trouble MEA between Student 9 and the researcher during the group presentation.

Student 9: : We found the difference in the electricity produced from landfill gas between 2019 and 2004. In 2019, electricity was 500.000. In 2004, electricity was 6.000. In 15 years, we found that 494.000 more electricity was produced by subtracting 6.000 by 500.000. To find out how much more electricity is produced in a year, we divide 494,000 by 15.

Researcher: You reused the method you used in the first question, am I correct?

Student 9: Yes, teacher. From here, we found 33.000. Then, we again added 33.000 one by one. We found the result as 698.000.

Researcher: What is 698.000?

Student 9: Electricity.

Researcher: What was asked in the question? The amount of trash or the amount of electricity?

Student 9: Immm…

Researcher: I think, you did not make any connection between the amount of trash and the amount of electricity in 2025. Please, revise your solutions.

The conversation above pointed out that the students solved the second question by using unit ratio. However, they did not make any connection between the trash and electrical energy produced by landfill gas. In other words, they just focused on electrical energy and did not find the amount of trash in 2025 to produce 650.000 MWh of electricity. The students in the fourth group did not make any revisions during the follow-up part and did not conclude the second question.

Researcher's account of Group 4's model of the Trash Trouble. Based on the answer of the 4th group, their solution was based on ratio and proportion. The reason is that they found the composed unit by proportioning the total amount of increase by the total number of years. The model that I derived as the researcher from the solution of the students in the fourth group for the first question is given in the expression below. As can be seen from the expression below, the students reached a unit ratio which was equal to amount of increase per year.

$$\frac{total\ amount\ of\ increase}{total\ number\ of\ years} = amount\ of\ increase\ in\ per\ year$$

In summary, if total amount of increase is proportioned by the total number of years, a unit ratio which corresponds to the amount of increase per year was obtained. The students reached this conclusion with mathematical thinking and proportional reasoning.

4.1.1.5 Solutions of the Group 5 in the Trash Trouble

Amount of trash in 2025. The students in the fifth group thought that they could solve the problem by using algebra. At first, they tried to examine the amount of increase between years. Then, they tried to find a pattern between the amounts of increases. They realized that there was approximately a 100.000 increase from 2008 to 2010, there was approximately a 200.000 increase from 2010 to 2012, there was approximately a 300.000 increase from 2012 to 2014. These increases continued in a pattern until 2018. Then, they continued this pattern and added the amount of increases to each year until 2025 as follows:

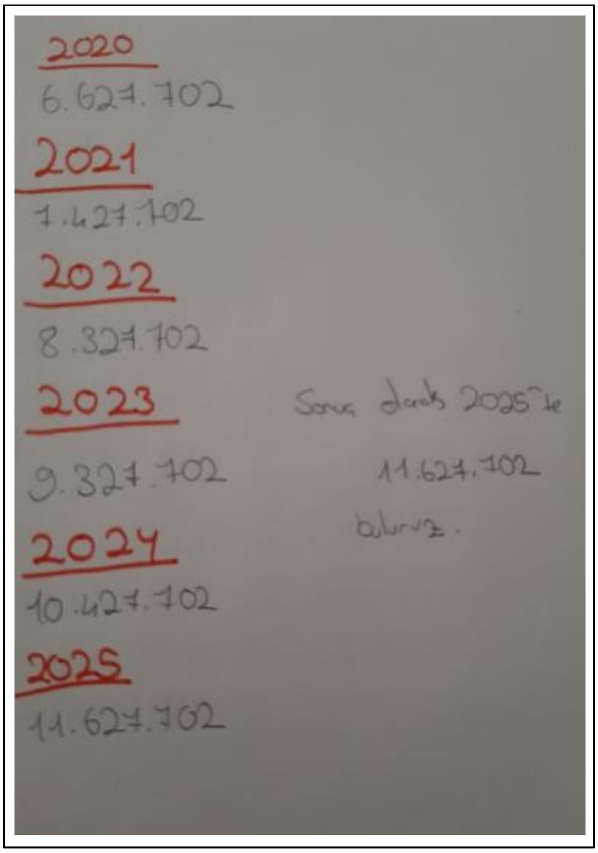


Figure 4.10 Group 5’s work in the Trash Trouble MEA

This figure shows that the students found that there would be 11.627.702 tons of trash in 2025. They determined a pattern that continued with 100.000 more increase than the previous year. They ignored the data of 2018 and 2019. They thought the pattern should continue as 700.000, 800.000, and so on. Thus, they added 700.000 to the data of 2019 and found the amount of trash in 2020 as 6.627.702. They continued with this pattern and added 12.000.000 tons of trash to 2024. Therefore, they found 11.627.702 tons trash for 2025.

Below is the conversation related to the students' solution for the first question of Trash Trouble MEA between Student 13 and the researcher during the group presentation.

> *Student 13:* We saw that from 2008 to 2010, the amount of trash increased approximately by 100.000. From 2010 to 2012, the amount of trash increased by approximately 200.000; from 2012 to 2014, approximately 300.000; from 2014 to 2015, approximately 400.000; from 2015 to 2016, approximately 500.000, and from 2016 to 2017, approximately 600.000. Then, we thought that there could be a pattern in the amount of increases.
>
> *Researcher:* Did you examine the years between 2017 and 2018, and 2018 and 2019?
>
> *Student 13:* Yes, but we ignored it since from 2017 to 2018 amount of trash increased to approximately 500.000, and from 2018 to 2019, there was a slight decrease. We thought that these two data would not be suitable for the rule of the pattern.
>
> *Researcher:* Okey, then?
>
> *Student 13:* Then, we added the amounts starting from 2019 until reaching 2025. Every year, we added the amount of trash which was 100.000 more than the previous year.
>
> *Researcher:* What does that mean?

Student 13: This means that from 2016 to 2017, the amount of trash increased by approximately 600.000. We added 100.000 to 600.000 and this is equal to 700.000. To find the amount of trash in 2020, we added 700.000 trash to the data of 2019. Similarly, for 2021, we added 800.000 to the data of 2020 that we had found. Like this, we found that in 2025, there would be 11.627.702 tons of trash.

The conversation above highlighted that students found 11.627.702 trash at the end. They found the amount of increase in each year by comparing the previous year. Then, they added the amount of increase to each year from the pattern that they generalized until they reach 2025. However, they did not examine the data for the years 2004 and 2006.

Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025. When works of the students in the fifth group were examined, it was seen that they did not do any work related to the second question. Student 11 stated that "We could not do the second question." As deduced from the Student 11's statement, the students did not do any work for the second question. They might have had problem with time and did not work on the second question. Thus, the students could not reach a conclusion for the second question.

Researcher's account of Group 5's model of the Trash Trouble. Based on the answer of the 5th group, their solution was based on pattern and generalization. The reason was that they determined a pattern and added the amount of trash to each year after 2019 based on this pattern. The model that I gathered as the researcher from the solution of the students in the fifth group for the first question is given in the algebraic expression below. As can be seen from the algebraic expression below, the students reached an algebraic pattern.

$$n, n+100.000, n+200.000, n+300.000, \ldots$$

Consequently, we can see a pattern of $n, n+100.000, n+200.000, n+300.000, \ldots$. It means that we could add to each year an amount of trash which was 100.000 more than the previous year. The students reached this conclusion with mathematical thinking and algebraic reasoning.

4.1.2 MEA 2 – Minimum Waste, Maximum Pencil Box

The second MEA was Minimum Waste, Maximum Pencil Box which consisted of two questions. The first question of the problem was related to developing a model to minimize the amount of waste materials when making the bottoms and sidewalls of a metal pencil box on a 100 cm x 100 cm square layer. In the first question, the students were expected to find how many pencil boxes they could place on the layer using the model that they developed. The second question of the problem was "Assuming that the pencil box's radius and height will not change, what should be the size of the square layer to get minimum amount of waste and maximum number of pencil boxes?" In the second question, the students were expected to find the size of the square layer without changing the sizes of the pencil box.

Mathematical learning residuals that I - as the researcher - expected from students was to draw the net of the cylinder, determine the radius/perimeter of the circle formed and the short and long sides of the rectangle formed after drawing the net of the cylinder. Furthermore, I expected them to use ratio and proportion to make smaller versions of the shapes and the layer while solving the problem. I expected this because they may have worked more easily with smaller shapes and layer. Environmental learning residuals that I - as the researcher - expected from the students was to realize the issue of trash/waste and to seek a solution for this issue.

4.1.2.1 Solutions of the Group 1 in the Minimum Waste, Maximum Pencil Box

Finding the Maximum Number of Pencil Boxes. The students in the first group thought that they could solve the first question by using ratio and proportion. They started to draw a cylinder and the net of the cylinder, showed the diameter, perimeter and height of the circle that can be viewed in Figure 4.11, and as represented in Figure 4.12, they showed the short and long sides of the rectangle.

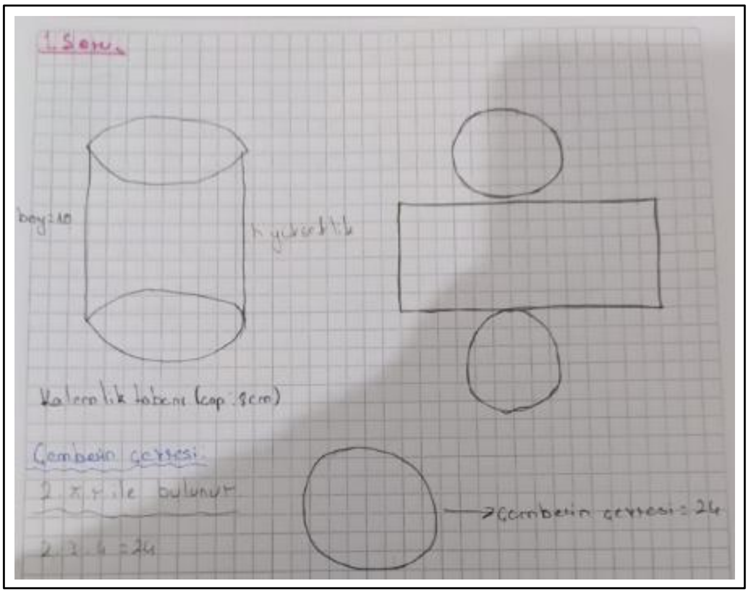


Figure 4.11 Group 1’s work to draw a cylinder and net of the cylinder in the Min. Waste, Max. Pencil Box MEA

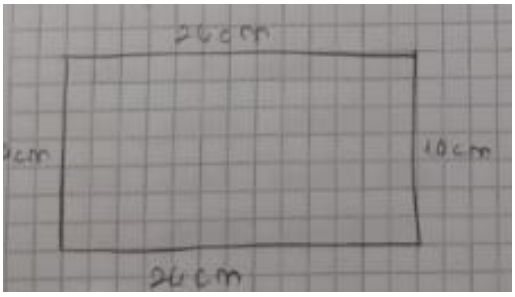


Figure 4.12 Group 1's work to determine the sizes of the rectangle formed when they drew the net of the cylinder in the Min. Waste, Max. Pencil Box MEA

As shown in Figure 4.11, the students in the 1st group drew a cylinder and the net of the cylinder. They found the perimeter of the circle formed when they drew the net of the cylinder from the $2.\pi.r$ formula as 24. They took radius of the circle as 4. Then, as Figure 4.12 presents, they determined the sizes of the rectangle formed when they drew the net of the cylinder. Since the perimeter of the circle is equal to the side of the rectangle surrounded by the circle, they wrote 24 cm to one of the sides of the rectangle. Since the height of the cylinder is equal to the side of the rectangle not surrounded by the circle, they wrote 10 cm to the other side of the rectangle.

Then, as shown in Figure 4.13 they made the layer, circle and rectangle smaller and found the sizes of the smaller shapes.

$\frac{100}{100} = \frac{10}{10}$ **Yani 100 ü 10 a böldük ve oran orantı kullandık.**

Daha sonra bu orantıyı kurduğumuz için elimizdeki her veriyi ona böldük.

ÇEMBERİN ÇAPI :
Normalde = 8 cm
Ona böldüğümüzde =0,8 cm

ÇEMBERİN YARIÇAPI :
Normalde = 4 cm
Ona böldüğümüzde =0,4 cm

DİKDÖRTGEN :
Normalde uzun kenar : 24 cm
Ona böldüğümüzde uzun kenar : 2,4

Normalde kısa kenar : 10 cm
Ona böldüğümüzde kısa kenar : 1 cm

Figure 4.13 Group 1's work to make the layer, circle and rectangle smaller in the Min. Waste, Max. Pencil Box MEA

It can be seen from Figure 4.13, that the students made the layer smaller. To do this, they divided each side of the square by 10. It means that they used $\frac{1}{10}$ ratio. Thus, they worked with a 10 cm x 10 cm square layer instead of a 100 cm x 100 cm square layer. Accordingly, they divided the radius and diameter of the circle and the short and long sides of the rectangle by 10. After all, they found the radius of the circle as 0,4 cm, the diameter of the circle as 0,8 cm, the long side of the rectangle as 2,4 cm and short side of the rectangle as 1 cm.

Then, they tried to place the rectangles on the layer. They realized that they could place 10 rectangles on the 10 cm x 10 cm square layer vertically, and 4 rectangles on the 10 cm x 10 cm square layer horizontally. Below is the explanation with regard to placement of the rectangles taken from the first group's written field notes.

> "If we think about it, 10 rectangles fit perfectly when we look vertically on a 10 cm x 10 cm square, when we look horizontally, 4 rectangles fit, and there is still little area."

As understood from the sentence above , the students found how many rectangles they could place on the layer vertically and horizontally without placing the circles.

After that, they thought that circle is an irregular shape in comparison with a rectangle or a square. For this reason, they tried to make a connection between the circles and rectangles. They realized that 3 circles could be placed in a rectangle and drew that. The figure below shows this placement.

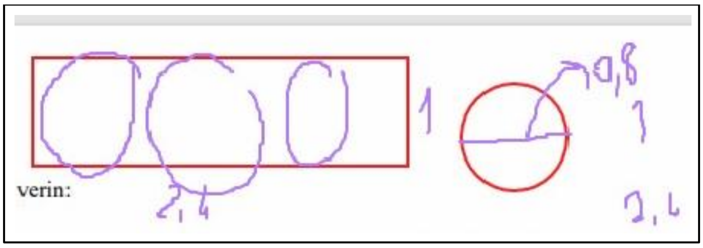


Figure 4.14 Group 1’s work to place three circles into a rectangle in the Min. Waste, Max. Pencil Box MEA

This figure shows that the students multiplied the diameter of a circle (0,8 cm) by 3 and found 2,4 cm to find the total length of the diameter of the 3 circles that they brought together (as tangential). The long side of the rectangle was 2,4 cm, and 2,4 cm was equal to the total length of the diameter of 3 circles. In addition, the short side of the rectangle was 1 cm, and 1 cm was smaller than the diameter of one circle. Thus, they could place 3 circles in a rectangle tangentially. This was important from the modeling perspective because they tried to construct a visual model by placing 3 circles in a rectangle.

Lastly, they realized that they could make 3 pencil boxes from 4 rectangles. This was because one circle and one rectangle are required to make a pencil box. If there will be 3 circles in a rectangle, there must be extra 3 rectangles for these circles. Previously, they found that they could place 10 rectangles vertically, and 4 rectangles horizontally. Since 3 pencil boxes were made with 4 rectangles in a row, and there were 10 rows, they found that they could place 30 pencil boxes on the 10 cm x 10 cm square layer by multiplying 3 by 10. Consequently, they found that they could place 300 pencil boxes on the 100 cm x 100 cm square layer multiplying 30 by 10. They multiplied 30 by 10 to reconstruct the layer since, at the beginning, they used $\frac{1}{10}$ ratio.

Below is the conversation related to the students' solution for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 1, Student 2 and the researcher during the group presentation.

> *Student 1:* There is a cylinder with a diameter of 8 cm and a height of 10 cm. When we draw the net of the cylinder, we have one rectangle and two circles. We know that the diameter is twice the radius. Thus, we found the radius as 4 by dividing 8 by 2. We found the circumference of the circle as 24 from the formula of $2.\pi.r$. In the question, there was a square layer of 100 cm x 100 cm. We made the layer smaller by using ratio and proportion. To do that, we divided 100 by 10 and accepted the layer as 10 cm x 10 cm.
>
> *Researcher:* Why did you use ratio and proportion?
>
> *Student 1:* Since this is a big layer, we made it smaller to solve the problem easier.
>
> *Researcher:* Okey, then?
>
> *Student 1:* Then, we found the short and long sides of the rectangle formed when we drew the net of the cylinder. We learned that the long sides of the rectangle are equal to the circumference of the circle. It means that they 24 cm. In addition, we learned that the short sides of the rectangle are equal to the height of the rectangle. It means they are 10 cm. Now, *Student 2* will continue.
>
> *Researcher:* Okey, Student 2 please continue.
>
> *Student 2:* Since we used ratio and proportion in the layer, we had to use this ratio in each shape. Firstly, the diameter of the cylinder was 8. We divided

it by 10 and found 0,8. Secondly, the long sides of the rectangle were 24 cm. We divided it by 10 and found 2,4. The short sides of the rectangle were 10 cm. We divided it by 10 and found 1. Since the short sides of the rectangle were 1 cm, we saw that we could place 10 rectangles on the 10 cm x 10 cm square vertically. We placed 4 rectangles on the 10 cm x 10 cm square layer horizontally since 4x2,4=9,6. The remaining parts of the layer would be wasted. In addition, we realized that we could place 3 circles in a rectangle. Then, we thought "why don't we place 3 circles in a rectangle." Then, we realized that we could make 3 pencil boxes from 4 rectangles.

Researcher: It means that one rectangle and one circle are required to make a pencil box. We can place 3 circles in a rectangle. We can make 3 pencil boxes from 4 rectangles.

Student 2: Yes. Then, we found how many pencil boxes we could make. We found that we could make 30 pencil boxes by multiplying 3 by 10 for the 10 cm x 10 cm square layer. Lastly, we found 300 pencil boxes by multiplying 30 by 10 for the 100 cm x 100 cm square layer.

Researcher: I have a question. Why did you multiply 30 by 10?

Student 2: We multiplied because the sizes of the original layer were 100 cm x 100 cm, and at the beginning, we made a smaller square. Thus, we thought that we had to reconstruct the square layer to its original size.

Researcher: Did you reconstruct the sizes of the other shapes – circle and rectangle?

Student 2: Nooo, we forgot.

Researcher: Okey, would you like to revise your solution?

Student 1/ Student 2: Yes.

The conversation above showed that the students found 300 pencil boxes at the end. However, it was understood that they forgot to reconstruct the rectangles while reconstructing layer. It means that while they enlarged the 10 cm x 10 cm square layer by the ratio of $\frac{1}{10}$, they did not enlarge the rectangles placed in the layer.

After they revised their solutions, the first group found the answer as 30 pencil boxes. Below is the conversation related to their solution after the revision of the first question of Minimum Waste, Maximum Pencil Box MEA between Student 2 and the researcher in the semi-structured interview.

> *Student 2:* We forgot to restore the circle and rectangle. Thus, we found the number of the pencil boxes incorrectly. Then, we revised our solution.
>
> *Researcher:* What did you do for revision?
>
> *Student 2:* We both enlarged the square layer and other shapes. Then, we realized that the number of the pencil boxes had to be the same on both a 10 cm x 10 cm square layer and a 100 cm x 100 cm square layer.
>
> *Researcher:* What is your answer after the revision?
>
> *Student 2:* 30 pencil boxes.

This conversation indicated that after the group presentation, the students revised their solution and reached 30 pencil boxes as the result. In other words, they both enlarged the layer and rectangles by the ratio of $\frac{1}{10}$.

Finding the Sizes of the Square Layer for Minimum Waste. Firstly, the students used ratio and proportion for the second question. They chose to enlarge the square layer multiplying the number of each shape by 2 which is given in Figure 4.15.

2.SORU :
10 x 10 cmlik dikdörtgene yataydan 4 dikeyden 10 dikdörtgen sığıyorsa bunu kafama göre aynı oranda artırıp azaltabilirim.

Örnek : dikey = 10 x 2 = 20 yatay = 4 x 2 = 8 istediğim sayıya bölüp çoğaltabilirim.

Figure 4.15 Group 1's work in the second part of the Min. Waste, Max. Pencil Box MEA

As shown in Figure 4.15, the students thought that they could enlarge or make smaller the layer by using ratio. They enlarged the 10 cm x 10 cm square layer by multiplying by 2. They also multiplied the number of rectangles in both row and column by 2.

Below is the conversation related to the students' solution for the second question of Minimum Waste, Maximum Pencil Box MEA between Student 2 and the researcher in the semi-structured interview.

> *Student 2:* We can place 4 rectangles horizontally and 10 rectangles vertically on 10 cm x 10 cm square layer. I could increase or decrease the rectangles and the layer at the same ratio. For example, we could place 4 rectangles horizontally, and we enlarged it by multiplying by 2. Thus, we found 8. We could place 10 rectangles vertically, and we also enlarged it by multiplying by 2. Thus, we found 20. Therefore, the number of pencil boxes increased, and the amount of waste also increased. In the question, the minimum amount of waste and the maximum number of pencil boxes were asked. Therefore, we could multiply or divide the ratio based on how much waste we wanted.
>
> *Researcher:* What will be the size of the square layer?
>
> *Student 2:* We should multiply it by 2 to fix the ratio. It means that the size of the square layer should be 200 cm x 200 cm since we enlarged it by

multiplying by 2. Actually, we could change the size of the square layer based on what was desired. If I aim to make the maximum number of pencil boxes, maybe we should not care about wasted materials. Moreover, even if the amount of waste changes, the amount remains the same as the ratio.

Researcher: As a result, what did you choose for the size of the square layer?

Student 2: 200 cm x 200 cm. We chose that because we thought the maximum number of pencil boxes would be more useful for the manufacturing company. We do not want to use a bigger layer since it would not be practical.

This exchange between the student and the researcher indicated that the students thought they could choose the sizes of the square layer based on their aim. They thought that if they enlarge the layer, they could make more pencil boxes, and that would be useful. Thus, they enlarged the layer of 100 cm x 100 cm by multiplying by 2 and determined it as 200 cm x 200 cm.

Researcher's account of Group 1's model of Minimum Waste, Maximum Pencil Box. Relying on the answer of the 1st group, their solution was based on ratio and proportion mathematically. The reason was that they divided each side of the square layer by 10 to work easily with a smaller layer. Respectively, they divided each side of the pencil box bottom (circle) and sidewall (rectangle) by 10 to keep the ratio of $\frac{1}{10}$. In addition, they used the right circular cylinder, rectangle, circle and square concepts and properties of these concepts while solving the problem. The model that I deduced as the researcher from the solution of the students in the first group is given in Figure 4.16. As can be seen from the figure below, the students reached a visual model by placing 3 circles, namely the pencil box bottoms, into a rectangle. They demonstrated 3 circles with 1 rectangle and 3 pencil boxes with 4 rectangles. Thereby, students placed rectangles into the square layer properly instead of placing both rectangles and circles.

3 circles = 1 rectangle

4 rectangles (3 circles and 3 rectangles) = 3 pencil boxes

Figure 4.16 Student-drawn model of students in the 1st group for Min. Waste, Max. Pencil Box MEA

Consequently, the net of a cylinder consists of two circles and one rectangle. If we take π as 3, three times the diameter of one circle is equal to one circle's perimeter from the perimeter formula of 2.π.r. In other words, 2.r in the formula is equal to the diameter of a circle, and 3(π).2r will be both equal to the diameter of 3 circles and the perimeter of one circle. At the same time, the perimeter of one circle is equal to the side of the rectangle surrounding the circle. Thus, 3 circles can be placed in the rectangle tangentially if their diameter is smaller than or equal to the other side of the rectangle not surrounded by the circle. In short, the students reached the conclusion shown in Figure 4.17 with geometric reasoning.

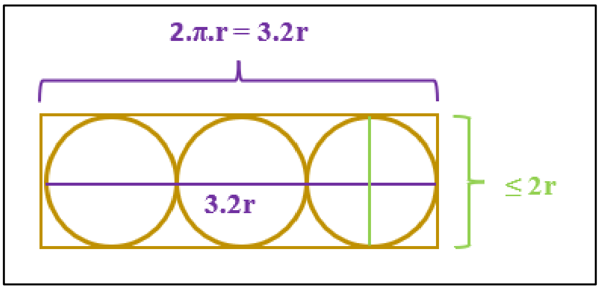


Figure 4.17 Researcher-generated model based on Group 1's way of thinking on Min. Waste, Max. Pencil Box MEA

In addition to the previous conclusion, the students reached another conclusion with proportional reasoning. This conclusion was that if they enlarge a shape with a specific ratio, they must enlarge other shapes corresponding with the main shape with the same ratio.

4.1.2.2 Solutions of the Group 2 in the Minimum Waste, Maximum Pencil Box

Finding the Maximum Number of Pencil Boxes. It was observed that the students in the second group had difficulty understanding the problem. At the beginning of the study, they drew a square, a rectangle and a circle which can be viewed in Figure 4.18.

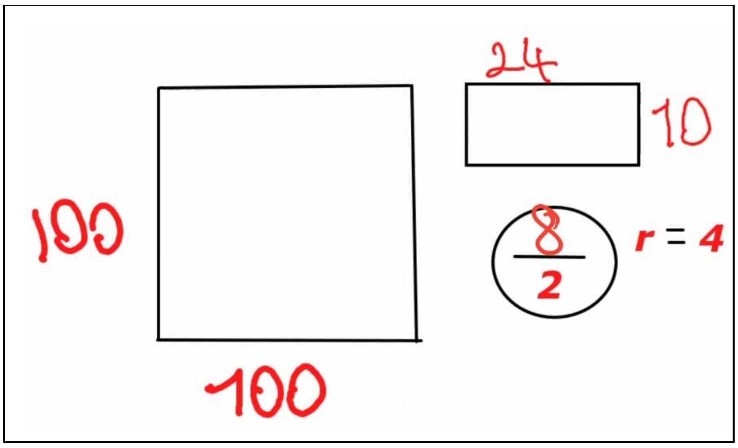


Figure 4.18 Group 2’s work to draw a cylinder and net of the cylinder in the Min. Waste, Max. Pencil Box MEA

From the figure above, we can see that the students in the 2nd group drew a square representing the 100 cm x 100 cm layer and drew the net of the cylinder – a rectangle and a circle – representing a pencil box bottom and sidewall. They wrote the short side of the rectangle as 24 and long side as 10. They also found the perimeter of the circle as 4 by dividing the diameter (8) of the circle by 2. Then, they tried to place the rectangles and circles in the layer as demonstrated in Figure 4.19.

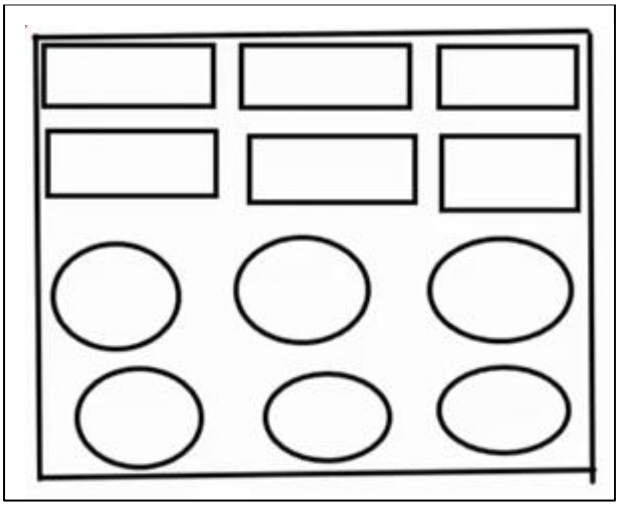


Figure 4.19 Group 2’s work to place the rectangles and circles in the Min. Waste, Max. Pencil Box MEA

This figure demonstrated that the students placed the rectangles and circles in the layer randomly without any computation. After placing the rectangles and circles in the layer randomly, the students did not do any work and did not reach any conclusion related to the number of the pencil boxes and a model.

Below is the conversation related to the students’ solution for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 3, Student 4, Student 5 and the researcher during the group presentation.

> *Student 5:* First of all, we drew a square like that. Then, we found the sizes of these shapes like our friends in the first group. Then, we drew six squares and six rectangles inside the square.
>
> *Researcher:* Why did you draw six squares and six rectangles?
>
> *Student 5:* Actually, we are not sure. We did not find the answer.

Student 3: Student 5 had some problems related to the Internet connection. For this reason, we could not work and find the answer.

Researcher: Okey, I want you to continue to work and revise your solution during the revision part.

Student 3, Student 4 and Student 5: Okey.

As understood, the students had some Internet connection problems, and they could not work effectively. They drew the net of the cylinder, determined the sizes of the shapes and drew six squares and six rectangles inside the square layer randomly. However, they did not conclude.

The students in the second group did not make any revisions during the follow-up part and did not conclude the first question.

Finding the Sizes of the Square Layer for Minimum Waste. When works of the students in the second group were examined, it was seen that they did not do any work related to the second question.

Below is the conversation related to the students' solution for the second question of Minimum Waste, Maximum Pencil Box MEA between Student 3, Student 5 and the researcher during the group presentation.

Researcher: What did you do about the second question?

Student 3: We did not find the answer to the second question.

Student 5: Actually, we did not understand the problem very well, and did not have enough time to work on the second question.

As deduced from the conversation above, the students did not understand the second question and had problem with time. Thus, the students could not conclude the second question.

Researcher's account of Group 2's model of Minimum Waste, Maximum Pencil Box. From the works of the 2nd group, it was seen that the students did not reach a mathematical model. The reason might be that they worked for 85 minutes during the modeling process part. This time was not enough for them to create a mathematical model.

4.1.2.3 Solutions of the Group 3 in the Minimum Waste, Maximum Pencil Box

Finding the Maximum Number of Pencil Boxes. At the beginning of the study, the students in the third group thought that they could solve the first question by using area or perimeter concepts. They started to draw the layer as can be seen in Figure 4.20, and find the sum of the perimeters of the rectangle and circle as illustrated in Figure 4.21.

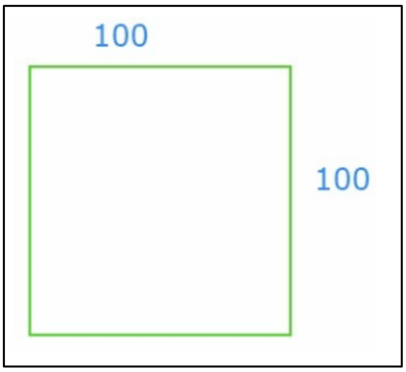


Figure 4.20 Group 3’s work to draw the layer in the Min. Waste, Max. Pencil Box MEA

10+24.2=20+48=68cm

68+24=92

Figure 4.21 Group 3's work to find the sum of the perimeters of rectangle and circle in the Min. Waste, Max. Pencil Box MEA

Figure 4.20 showed 100 cm x 100 cm square layer, and Figure 4.21 indicated that they found the perimeters of the rectangle and circle formed when drawing the net of the cylinder. They found the perimeter of the rectangle by multiplying 10 and 24 by 2 and adding 20 and 48. They also found the perimeter of the circle as 24 like the other groups. Lastly, they found the perimeter of a pencil box bottom and sidewall (a circle and a rectangle) as 92 by adding 68 and 24.

After finding the perimeter of a pencil box bottom and sidewall, the students in the third group thought that they would not be able to reach the solution with the perimeter concept. Then, they tried to place the rectangles on the 100 cm x 100 cm square layer without placing the circles. They thought that they could place 10 rectangles vertically and 4 rectangles horizontally. Starting from this point of view, they thought that 40 rectangles could be placed by multiplying 10 by 4. Since there would be wasted materials whose sizes were 4 cm x 100 cm, they thought that extra 4 rectangles could be placed. Thus, they found the answer as 44 pencil boxes.

After finding the answer of 44, they realized that they ignored the circles – pencil box bottoms. Then, they combined one rectangle and one circle and found the length of this model as 32 by adding the long side of the rectangle (24) and the diameter of the circle (8). This was important from the modeling perspective because they tried to construct a visual model by combining one rectangle and one circle in a rectangle. Then, they placed 30 of this model on the layer. Lastly, they thought that they could place 3 extra pencil boxes. (See Figure 4.22.)

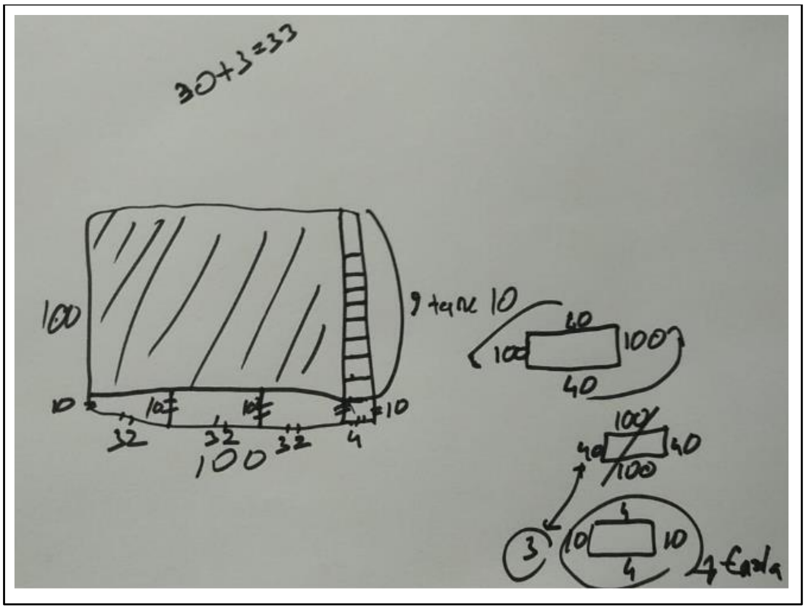


Figure 4.22 Group 3’s work to place the rectangles and circles in the Min. Waste, Max. Pencil Box MEA

The students tried to place the rectangles whose long side was 32 cm and short side was 10 cm on the layer which is displayed in the figure above. They placed 10 rectangles horizontally and 3 rectangles vertically. In total, they placed 30 pencil boxes. They found the answer as 33 by thinking that 3 more pencil boxes could be placed in the remaining area.

Below is the conversation related to the students’ solution for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 8 and the researcher during the group presentation.

> *Student 8:* Teacher, we tried three times for the first question. Firstly, we found the perimeter of the rectangle and circle. It means that we found the perimeter of a pencil box bottom and sidewall. Then, we did not continue from that point since we thought that we would not be able to find the answer

from the perimeter. Secondly, we tried to place just the sidewalls of the pencil box ignoring the bottoms of the pencil box – this was incorrect. When we did this, the answer was 44.

Researcher: How did you find the answer of 44?

Student 8: We placed 10 rectangles completely in vertical position since the short side of the rectangle was 10 cm, and the side of the square was 100 cm. It means that from 100:10, 10 rectangles could be placed vertically. In addition, we placed 4 rectangles in one row horizontally. That is because 4 times 24 – the long side of the rectangle – is equal to 96 cm. There is 4 cm extra. In total, we placed 40 rectangles by multiplying 4 by 10. Furthermore, we thought that we could place 4 rectangles on the extra area – the waste area. Thus, our answer was 44.

Researcher: Okey, go on.

Student 8: As I said before, we ignored the bottoms of the pencil box . Thirdly, we added 8 (the diameter of the circle) to the 24 (the long side of the rectangle) and found 32 cm.

Researcher: It means that since a pencil box consists of one circle and one rectangle, you combined them, and you constructed a rectangle whose long side is 32 cm and short side is 10 cm.

Student 8: Yes. Normally, according to you, the answer is 30 by multiplying 10 by 3 but we added the extras and found the answer as 33.

Researcher: How were you sure that 3 extra pencil boxes could be placed?

Student 8: After all, we can cut that material and create a rectangle.

Researcher: I am asking again, are you sure about the 3 extra pencil boxes?

Student 8: No teacher, if you allow us, we can revise that part.

Researcher: Of course, you can.

The conversation above demonstrates that the students reached 33 pencil boxes at the end. However, it was seen that they did not make an exact computation to place 3 pencil boxes on the remaining waste area.

After they revised their solutions, the third group found the answer as 31 pencil boxes. Below is the conversation related to their solution after revision for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 8 and the researcher in the semi-structured interview.

> *Student 8:* Teacher, we had a remaining area of a rectangle with the size of 4 cm x 100 cm. We divided that area into four rectangles. Three rectangles are equal, and their sizes are 4 cm x 32 cm. There is one more area whose sizes are 4 cm x 4 cm. Then, we combined three of these equal rectangles and constructed a rectangle of 32 cm x 12 cm . Since we can make a pencil box from a rectangle of 32 cm x 10 cm, we can also make an extra pencil box from a rectangle of 32 cm x 12 cm. So, we found the answer as 31.
>
> *Researcher:* Okey, what did you do with the 4 cm x 4 cm area?
>
> *Student 8:* Nothing, that area is waste material since with that area we cannot make a pencil box.

This conversation showed that after the group presentation, the students revised their solution and reached 31 pencil boxes as result. In other words, they placed 30 pencil boxes on the layer and made one extra pencil box from the waste area.

Finding the Sizes of the Square Layer for Minimum Waste. The students in the third group thought that they could change the sizes of the pencil box instead of changing the sizes of the layer to get the maximum number of pencil boxes and the minimum amount of waste. They changed the height of the pencil box without changing the radius of the bottom of the pencil box.

Below is the conversation related to students' solution for the second question of Minimum Waste, Maximum Pencil Box MEA between Student 6, Student 7 and the researcher during the group presentation.

Student 7: We tried to change the height of the sidewall of the pencil box . When we did this, we made sure that the height was not less than 5 cm because of its usefulness. In addition, we thought that the height should be less than 10 cm to make more pencil boxes.

Researcher: Actually, according to the question, you should change the sizes of the layer. However, your friends thought that they could make more pencil boxes by changing its height instead of changing the sizes of the layer.

Student 7: We thought that the height should be between 5 and 10 cm. Then, we took the height as 8 and found the perimeter of the sidewall of the pencil box. To do that, we multiplied 24 by 8 and found 192.

Researcher: : Is 192 the perimeter?

Student 7: Yes, it is the perimeter.

Student 6: No, it should be the area. Sorry, teacher.

Researcher: Okey, go on.

Student 7: We also found that the circle's area as 48. We found the total area for one pencil box as 240. We tried this process by taking the height as 9. When we took the height as 8, we got more pencil boxes. Consequently, we chose the height as 8 cm.

As understood from this conversation, the students tried to find a suitable height for the maximum number of pencil boxes without changing the sizes of the bottom of the pencil box and the layer. To do that, they used the area concept. Although they used the area concept, they thought of it as the perimeter. It can be said that they had a misconception related to the definition of area and perimeter concepts. On one hand, they ignored the amount of waste materials. Furthermore, they did not use exact computation while finding the number of the pencil boxes with different heights.

Researcher's account of Group 3's model of Minimum Waste, Maximum Pencil Box. Depending on the answer of the 3rd group, they solved the problem geometrically. They used right circular cylinder, rectangle, circle and square concepts and properties of these concepts while solving the problem. The model that I concluded as the researcher from the solution of the students in the third group is demonstrated in Figure 4.23. As can be seen from the figure below, the students reached a visual model by combining a rectangle and a circle tangentially to construct a longer rectangle. The longer rectangle represents one pencil box. Thereby, the students placed the rectangles into the square layer properly instead of placing both rectangles and circles as the students in the 1st group did .

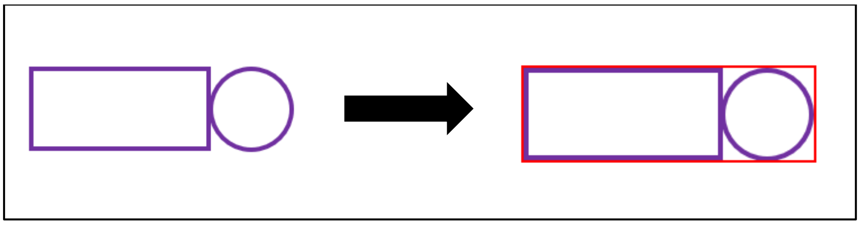


Figure 4.23 Student-drawn model of the students in the 3rd group for Min. Waste, Max. Pencil Box MEA

Hence, the net of a cylinder consists of two circles and one rectangle. If we take π as 3, three times the diameter of one circle is equal to the diameter of one circle which is also equal to the side of the rectangle surrounding the circle. We can combine one of these circles and this rectangle tangentially, and then we can place them in a longer rectangle tangentially if the circle's diameter is smaller than or equal to the other side of the rectangle not surrounding the circle. In this case, the long side of the longer rectangle will be equal to 8r. In brief, the students reached the conclusion indicated in Figure 4.24 with geometric reasoning.

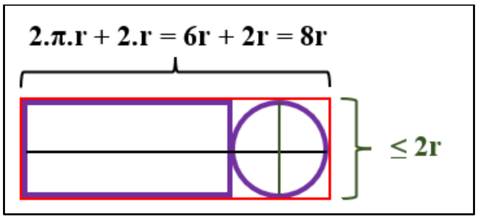


Figure 4.24 Researcher-generated model based on Group 3's ways of thinking on Min. Waste, Max. Pencil Box MEA

4.1.2.4 Solutions of the Group 4 in the Minimum Waste, Maximum Pencil Box

Finding the Maximum Number of Pencil Boxes. Similar to the students in the 1st group, those in the 4th group thought that 100 cm x 100 cm layer was too big and tried to make the layer smaller by using a specific ratio. In addition, they thought that they could solve the problem by using the area concept. At first, they drew a square layer by making it smaller and found its area as follows:

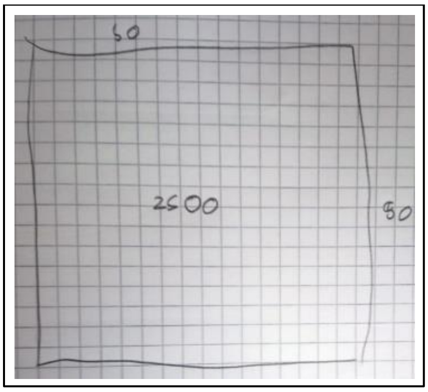


Figure 4.25 Group 4's work to draw and make the layer smaller in the Min. Waste, Max. Pencil Box MEA

In Figure 4.25 there was clear that students drew a square representing the layer. They used $\frac{1}{2}$ ratio, made the layer smaller and found each side of the square layer as 50. Then, they found the area of the layer as 2500.

Figure 4.26 indicates that they drew the rectangle smaller, found the perimeter of the smaller circle and the area of the smaller rectangle, and added them.

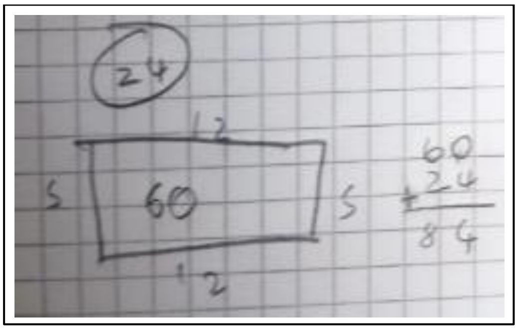


Figure 4.26 Group 4's work to find perimeter of the smaller circle and area of the smaller rectangle in the Min. Waste, Max. Pencil Box MEA

This figure shows that they drew a rectangle and showed its long side as 12 and its short as 5 by using ratio of $\frac{1}{2}$. Then, they found the area of the rectangle as 60. In addition, they found the perimeter of the circle as 24 like the other groups. However, they did not use $\frac{1}{2}$ ratio to make the circle smaller. In other words, while making the layer and the rectangle smaller, they did not make the circle smaller. Then, they added 60 and 24 and found the sum as 84.

Lastly, they divided 2500 by 84, and found 30. Then, they multiplied 30 by 2 and found the answer as 60 as shown in Figure 4.27.

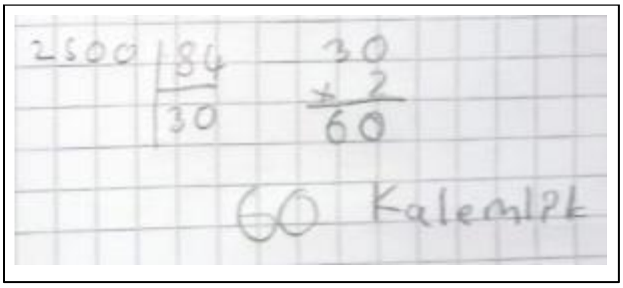


Figure 4.27 Group 4's work to find number of the pencil boxes in the Min. Waste, Max. Pencil Box MEA

Based on the data from this figure, it can be said that the students divided the area of the square layer by the sum of the perimeter of the circle and the area of the rectangle and found 30. They reconstructed the layer and other shapes by multiplying by 2 and found that 60 pencil boxes could be placed on the original layer.

Below is the conversation related to students' solution for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 9, Student 10 and the researcher during the group presentation.

> *Student 10:* We firstly made the layer smaller. I mean, we made the sizes of the layer 50 cm x 50 cm. Then, we drew a smaller rectangle and determined its size. Since its short side was 10 cm, we divided it by 2 and found 5 cm. I mean, the short side of the smaller rectangle was 5 cm. Similarly, the long side of the smaller rectangle was 12 cm. Then, we found the perimeter of the circle and rectangle. The perimeter of the circle was 24, and the perimeter of the rectangle was 60. Then, we added 60 and 24 and found the sum as 84.
>
> *Researcher:* Just a second. How can the perimeter of a rectangle be found?

Student 9: Sorry, teacher. We made a mistake. 60 is not the perimeter, it is the area.

Researcher: Yes. The perimeter of a rectangle is equal to the sum of all sides of the rectangle. The area of a rectangle is equal to the multiplication of its long side and its short side. Go on, please.

Student 10: Then, we divided 2500 by 84, and found 30 pencil boxes but the answer is 60.

Researcher: What do 2500 and 84 mean?

Student 10: Area of the layer and area of a pencil box. We divided it since we tried to find how many pencil boxes could be placed on the layer.

Researcher: Okey, but you divided the area of the layer by the sum of the area of the rectangle and perimeter of the circle. You did not divide the area of the layer by the area of a pencil box.

Student 9: Teacher, we made a lot of mistakes. In addition, we did not reconstruct the other shapes at the end like *Student 1* and *Student 2*. Can we revise our solution too?

Researcher: Yes, of course.

The conversation above shows that the students found 60 pencil boxes at the end. However, it was seen that they had some misconceptions. Firstly, they had a misconception related to the area and perimeter of the rectangle. Secondly, they forgot to reconstruct other shapes while reconstructing the layer. It means that while they enlarged the 50 cm x 50 cm square layer by the ratio of $\frac{1}{2}$, they did not enlarge other shapes.

After they revised their solutions, the fourth group found the answer as 34 pencil boxes. Below is the conversation related to their solution after revision for the first question of Minimum Waste, Maximum Pencil Box MEA between *Student 9* and the researcher in the semi-structured interview.

Student 9: Teacher, we corrected our mistakes and found the answer as 34 pencil boxes.

Researcher: How did you find it?

Student 9: We did not change the ratio of $\frac{1}{2}$. We worked with smaller shapes again. We already found the area of the layer as 2500 and the area of the rectangle as 60. We also found the area of the circle as 12 from the formula of $\pi.r^2$. Here, r is 2 since we used $\frac{1}{2}$ ratio. We added 60 and 12 and found 72. Then, we divided 2500 by 72 and found 34,72 approximately. We did not make any rounding because we thought that there cannot be 35 pencil boxes. Thus, we found that 34 pencil boxes can be placed on the layer.

This conversation showed that after the group presentation, the students revised their solution and reached 34 pencil boxes as the result. In other words, they both enlarged the layer and other shapes by the ratio of $\frac{1}{2}$. In addition, they corrected their misconceptions related to area and perimeter concepts.

Finding the Sizes of the Square Layer for Minimum Waste. When the fourth group's solution for the second question was examined, it was seen that they used ratio and proportion. Figure 4.28 represents that, similar to the first group's answer, they chose to enlarge the square layer by multiplying by 2.

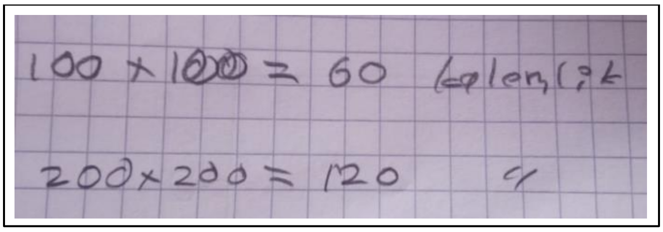


Figure 4.28 Group 4's work in the second part of the Min. Waste, Max. Pencil Box MEA

As Figure 4.28 shows, the students enlarged the 100 cm x 100 cm square layer by multiplying by 2. They multiplied the number of pencil boxes by 2 and found 120 pencil boxes. That is to say, they thought that if sizes of the layer are doubled, the number of pencil boxes is doubled.

Below is the conversation related to the students' solution for the second question of Minimum Waste, Maximum Pencil Box MEA between *Student 10* and the researcher during the group presentation.

> *Student 10:* Teacher, we thought that we should enlarge the layer to make more pencil boxes. We can choose any numbers for it. We chose 2. Since we made 60 pencil boxes with a 100 cm x 100 cm layer, we can make 120 pencil boxes with a 200 cm x 200 cm layer.
>
> *Researcher:* It means that you chose the layer of 200 cm x 200 cm.
>
> *Student 10:* Yes.

It can be understood from the conversation that the students thought that they could choose any size for the square layer. They thought that if they enlarge the layer, they could make more pencil boxes. Thus, they enlarged the layer of 100 cm x 100 cm by multiplying it by 2 and determined it as 200 cm x 200 cm.

Researcher's account of Group 4's model of Minimum Waste, Maximum Pencil Box. The solution of the 4^{th} group was based on ratio and proportion and area mathematically. The reason for the use of ratio and proportion was that they divided each side of the square layer by 2. It means that they used the ratio of $\frac{1}{2}$. The reason for the use of area was that they divided the area of the layer by the area of a pencil box to find how many pencil boxes could be made from the layer. In addition, they used right circular cylinder, rectangle, circle and square concepts and the properties of these concepts while solving the problem. The model that I inferred as the researcher from the solution of the students in the fourth group is given in the expression below. As can be seen from the expression below, the students reached a formula to find the total number of pencil boxes.

$$\frac{area\ of\ the\ layer}{area\ of\ a\ pencil\ box} = number\ of\ pencil\ boxes$$

This result indicates that if we divide the area of the layer by the total area of the materials which was used to make a pencil box, we get the number of pencil boxes which could be made from the layer. The students reached this conclusion with mathematical thinking and reasoning. Furthermore, the students reached the conclusion of if they enlarged a shape with a specific ratio, they must enlarge other shapes corresponding with the main shape with the same ratio with proportional reasoning like the student in the 1st group .

4.1.2.5 Solutions of the Group 5 in the Minimum Waste, Maximum Pencil Box

Finding the Maximum Number of Pencil Boxes. The students in the fifth group thought that they could solve the first question by using ratio and proportion similar to the students in the first and fourth groups. At first, they drew a rectangle and a circle when they drew the net of the cylinder. Then, they used $\frac{1}{10}$ ratio like the first group, and found the sizes of the smaller shapes as can be seen in Figure 4.29.

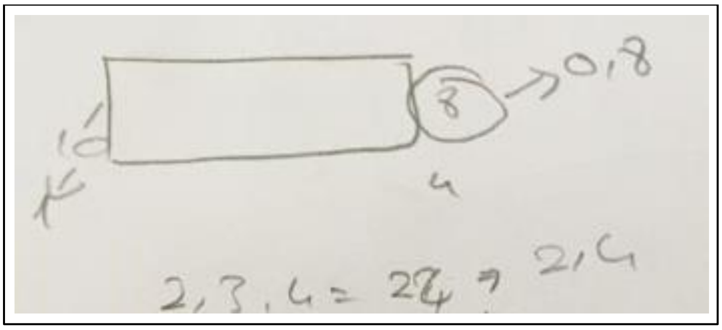


Figure 4.29 Group 5's work to find sizes of the smaller shapes in the Min. Waste, Max. Pencil Box MEA

From the figure above, it is clear to see that the students found the perimeter of the circle as 24. Since they used ratio of $\frac{1}{10}$, they found the perimeter of the smaller circle as 2,4 and found diameter of the smaller circle as 0,8. In addition, they found the short side of the smaller rectangle as 1.

After finding the sizes of the smaller shapes, they tried to place the rectangles on the layer. Figure 4.30 shows that they thought they could place three rectangles and three circles in a row horizontally and made computation related to this.

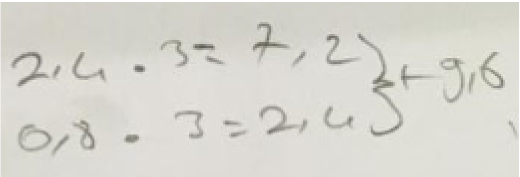


Figure 4.30 Group 5’s calculations of the number of pencil boxes in the Min. Waste, Max. Pencil Box MEA

Figure 4.30 indicates that the students tried to find whether three pencil boxes could be placed horizontally. To do this, they multiplied 2,4 by 3 and found 7,2. They multiplied 0,8 by 3 and found 0,4. Then, they added 2,4 and 7,2 and found the sum as 9,6.

Figure 4.31 reveals that they realized they could place 10 rectangles on the 10 cm x 10 cm square layer vertically.

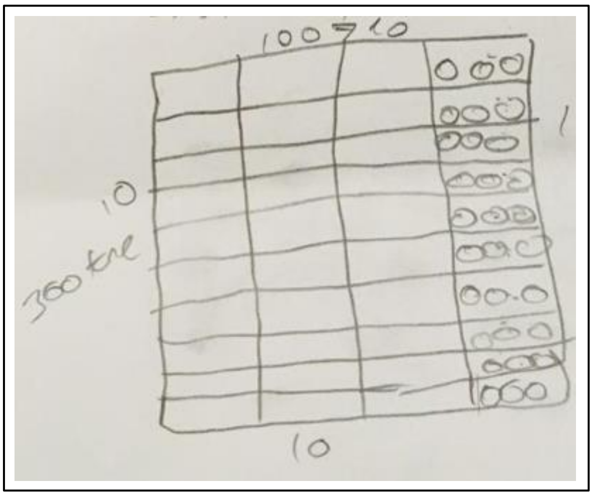


Figure 4.31 Group 5's work to place the pencil boxes in the Min. Waste, Max. Pencil Box MEA

Using the data from Figure 4.31, they placed three rectangles and three circles horizontally since 9,6 was smaller than 10 which was the side of the layer. In addition, they placed ten rectangles vertically since the short side of the rectangle was 1 cm and each side of the rectangle was 10 cm.

Similar to the first group, they found that they could place 300 pencil boxes on the 100 cm x 100 cm square layer by multiplying 30 by 10. They multiplied 30 by 10 to reconstruct the layer since they used $\frac{1}{10}$ ratio at the beginning.

Below is the conversation related to students' solution for the first question of Minimum Waste, Maximum Pencil Box MEA between Student 14 and the researcher during the group presentation.

> *Student 14:* We learned that the net of the cylinder consists of two circles and one rectangle. Since the top of our pencil box is open, there should be one circle and one rectangle. Then, we found the perimeter of the circle as 24. We found r as 4 since the diameter is 8 in our problem. The short side of

the rectangle is 10 cm. Then, we divided all numbers by 10 because we chose the layer as 10 cm x 10 cm.

Researcher: What does all numbers mean? Why did you choose the layer as 10 cm x 10 cm?

Student 14: All numbers means the long and short side of the rectangle and the diameter of the circle. We divided all these sizes by 10 because we chose the layer as 10 cm x 10 cm. We chose the layer as 10 cm x 10 cm because in the problem, the layer is 100 cm x 100 cm. It was too big. We thought that we can place and draw rectangles and circles on a 10 cm x 10 cm layer easily.

Researcher: Okey, you used ratio and proportion. You made the rectangle, circle and layer smaller by using ratio of $\frac{1}{10}$. Then?

Student 14: Yes. Then, we tried to place the rectangles and circles on the layer. We thought that we could place 3 rectangles horizontally in a row. To prove this, we multiplied 3 by 2,4 – 3 x long side of the rectangle and found 7,2. Then, we multiplied 3 by 0,8 since the diameter of the circle was 0,8.

Researcher: Why did you use 3 circles?

Student 14: Because we placed 3 rectangles on the layer. We needed 3 circles and 3 rectangles to make 3 pencil boxes.

Researcher: Okey.

Student 14: Then, 3x0,8 is equal to 2,4 cm. We added 2,4 and 7,2 and found 9,6. 9,6 is the area that we used in the square layer. There are 10 rectangles vertically and 3 rectangles horizontally. In total, there are 30 rectangles and 30 circles. It means 30 pencil boxes. When we make the layer 100 cm x 100 cm, there will be 300 rectangles and 300 circles. It makes 300 pencil boxes but we noticed that we did not enlarge the rectangles and circles while listening to *Student 1* and *Student 2*'s presentation.

Researcher: Okey, you can revise your solution.

Student 14: Yes.

This conversation between the student and the researcher shows that the students in the 5th group found 300 pencil boxes at the end. However, similar to the 1st group's misconception, they forgot to reconstruct the squares and circles while reconstructing the layer. After they revised their solutions, the 5th group found the answer as 30 pencil boxes.

Finding the Sizes of the Square Layer for Minimum Waste. When the fifth group's solution for the second question was examined, it was seen that they did not change the sizes of the layer, and wanted to use a 100 cm x 100 cm square layer.

Below is the conversation related to students' solution for the second question of Minimum Waste, Maximum Pencil Box MEA between Student 14 and the researcher in the semi-structured interview.

Student 14: We again chose a 100 cm x 100 cm layer since the amount of waste was less. When we placed the circles and rectangles, there remained a waste area of 0,4 horizontally. This was tiny. There was already minimum amount of wasted area which was targeted in the question.

Researcher: I t means you said that there is already minimum amount of waste materials.

Student 14: Yes, teacher. We thought that if we enlarge the layer, the amount of waste will increase.

As deduced, the students thought that they could choose a square layer of with same size since when three circles and three rectangles were placed, there was less amount of waste. Therefore, they did not feel the need to change the sizes of the square layer.

Researcher's account of Group 5's model of Minimum Waste, Maximum Pencil Box. Similar to the 1st group's answer, the solution of the students in the 5th group was based on ratio and proportion mathematically because they used the ratio of $\frac{1}{10}$. In addition, they used right circular cylinder, rectangle, circle and square concepts and the properties of these concepts while solving the problem. The model that I discovered as the researcher from the solution of the students in the fifth group is given in Figure 4.32. As can be seen from the figure below, the students developed a visual model by placing 3 rectangles and 1 more rectangle for 3 circles side by side in a row.

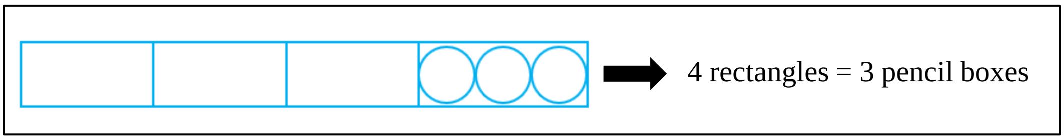


Figure 4.32 Student-drawn model of the students in the 5th group for the Min. Waste, Max. Pencil Box MEA

To conclude, the net of a cylinder consists of two circles and one rectangle. If π is taken as 3, three times the diameter of one circle is equal to the side of the rectangle surrounded by the circle. In other words, 6r is equal to the long side of the rectangle. Since 3 rectangles are placed side by side horizontally, their length is equal to 3 times 6r, which is 18r. The diameter of three circles is 3.2r, 6r. If the circle's diameter is smaller than or equal to the other side of the rectangle not surrounded by the circle, three circles can be placed into one rectangle. Thereby, a model consisting of 4 rectangles whose long side is 24r can be constructed. Concisely, the students reached the conclusion shown in Figure 4.34 using geometric reasoning.

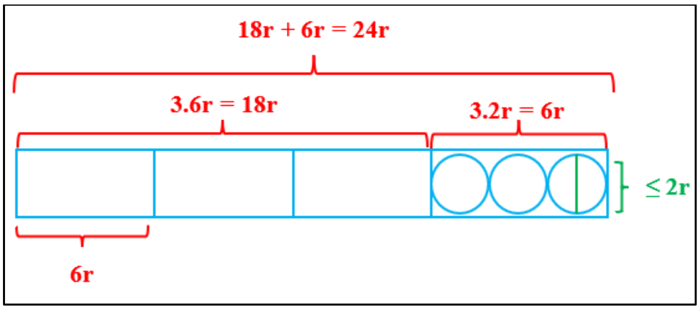


Figure 4.33 Researcher-generated model based on Group 5's ways of thinking for Min. Waste, Max. Pencil Box MEA

Furthermore, the students reached another conclusion that if they enlarge a shape with a specific ratio, they must enlarge other shapes corresponding to the main shape with the same ratio as the students in the 1^{st} and 4^{th} group did.

4.2 Mathematical Learning Residuals

When all groups' solutions to Trash Trouble were examined, it can be said that all five groups solved the problem by using different ways. To illustrate, the 1^{st} group used pattern and generalization while the 3^{rd} group used arithmetic average concepts. Although they solved the problem by using different ways, there were some points in common in their solutions. For example, most of the groups tried to find the amount of increase/decrease between the amount of trash in years. Cross-comparison of the characteristics of the models of five groups for Trash Trouble is set out in Table 4.1 below.

Table 4.1 Cross-comparison of the characteristics of the models of five groups for the Trash Trouble MEA

Groups	**Amount of trash in 2025**	**Amount of trash to produce 650.000 MWh of electricity from landfill gas in 2025**
Group 1	Finding the amount of increase/decrease between the amount of trash Generating an algebraic pattern	Proportioning the amount of trash to the amount of electrical energy and using arithmetic average
Group 2	Examining the total change in the amount of trash between 2014 and 2015 Making a guess without any computation	-
Group 3	Finding the average amount of trash per year	Proportioning the amount of electrical energy to the amount of electrical energy and proportioning the amount of trash to the amount of trash
Group 4	Finding the difference between the amount of trash in 2019 and the amount of trash in 2004 Finding the amount of increase in the trash per year	Finding the amount of increase in the electricity per year
Group 5	Examining the amount of increase between years Generating a pattern	-

Table 4.1 presents that all five groups tried to develop a model to solve the first part of Trash Trouble. However, while the 1st, 3rd and 4th groups developed a model, the

2nd and 5th groups did not develop a model for the second part of the problem. As the researcher, the model that I saw as more satisfying was the first group's model since they examined the amount of increase and determined an algebraic pattern. In addition, since the amount of increases was different, the 1st group's model was strong in terms of determining the amount of trash in the targeted year. As the researcher, the model that I saw as the least strong was the second group's model because they just made a guess without any mathematical thinking and reasoning.

Group 1 used pattern and generalization for the first part while using ratio and proportion and the arithmetic average for the second part. Thus, it can be said that their solution approaches were not exactly related. Group 3 used the arithmetic average for the first part and used build-up strategy (between ratio model) for the second part. While finding the average amount of trash per year, they proportioned the total amount of trash to the total number of years. Consequently, their approaches in the first and second parts of the problem were related. Group 4's approaches were also related since they used unit ratio in both parts. Therefore, it can be said that Group 3 and 4's approaches could have helped them develop a solution in the second part. For Group 2, their approach of making a guess without any computation in the first part of the problem could have prevented them from developing a solution in the second part. Similar to the 2nd group, Group 5 did not develop a model for the second part. The reason might be that they could not make a relation between trash and electricity.

In a similar vein, there were some points in common based on the groups' solutions for Minimum Waste, Maximum Pencil Box. For instance, all groups tried to draw the net of the cylinder, and the 1st, 4th and 5th groups used ratio and proportion. Moreover, the 1st, 3rd and 5th groups placed the nets by decomposing the shapes, while the other groups placed the nets without decomposing the shapes. Cross-comparison of the characteristics of the models of five groups for Minimum Waste, Maximum Pencil Box is shown in Table 4.2 below.

Table 4.2 Cross-comparison of the characteristics of the models of five groups for the Minimum Waste, Maximum Pencil Box MEA

Groups	**Finding the maximum number of pencil boxes**	**Finding the sizes of the square layer for minimum waste**
Group 1	Attending to the net of the shape Finding the ratio of sizes of the layer to sizes of the pencil box Aligning the nets by decomposing the shapes (placing three circles into a rectangle)	Doubling the sizes of the square layer
Group 2	Attending to the net of the shape Aligning the nets without decomposing the shapes	-
Group 3	Aligning the nets by decomposing the shapes (placing one circle and one rectangle next to each other)	Changing the height of the pencil box without changing its radius and the sizes of the layer
Group 4	Finding the ratio of the sizes of the layer to the sizes of the pencil box Area-based solution Aligning the nets without decomposing the shapes	Doubling the sizes of the square layer
Group 5	Finding the ratio of the sizes of the layer to the sizes of the pencil box Aligning the nets by decomposing the shapes (placing three circles and three rectangles next to each other)	Not changing the sizes of the square layer

Table 4.2 illustrates that all of five groups tried to develop a model to solve the first part of Minimum Waste, Maximum Pencil Box. Nevertheless, the second group did not develop a model to solve the second part of the problem. As the researcher, the model that I saw as more satisfying was the first group's model since, they constructed the model by placing 3 circles into 1 rectangle and made a more regular layout with less waste on the layer. As the researcher, the model that I saw as the least strong was the second group's model because they just placed the rectangles and circles in the layer randomly. They did not construct an actual model.

For groups 1 and 4, the approach of using ratio and proportion to make the layer and other shapes smaller in the first part helped them solve the second part of the problem. They also used ratio and proportion and enlarged the layer in the second part. That is to say, their approaches in the first and second parts of the problem were related. For group 2, their approach of random placement and not reaching the exact result in the first part of the problem could have limited them in terms of developing a solution in the second part. Group 3 reached solution unrelated to what the second question had asked for, and Group 5 did not change the sizes of the square layer for the second question. It means that these two groups' approaches in the second question were not related to their approaches in the first question.

4.3 Environmental Learning Residuals

When written works, letters, presentations, semi-structured interviews and post-activity participant forms of all students were examined, learning residuals of students related to environmental issues were grouped under two themes for both MEAs. These themes raised students' awareness about the issue in two ways (1) understanding a local situation and (2) thinking about action strategies.

4.3.1 Raising awareness about the issue: Understanding a local situation

The students learned information about the excess amount of trash and inadequacy of recycling facilities for the Trash Trouble MEA. Below are some examples of the answers given by the students related to this theme.

> "Turkey ranks number two in terms of the amount of trash and cannot recycle most of this garbage." (Student 2's answer in the post-activity participant form)
>
> "I was aware that there is too much trash in Istanbul and that even one person can produce thousands of trash." (Student 1's answer in the post-activity participant form)
>
> "I realized how much human beings pollute the environment based on the amount of trash." (Student 9's answer during the semi-structured interview)
>
> "This is just the amount of trash in Istanbul, if we add up the amount of trash in Turkey, it means there is too much trash in our country." (Student 11's answer during group presentations)
>
> "We realized how much trash we produce." (Student 6's answer in the post-activity participant form)
>
> "I learned there is too much trash in Istanbul, but there are not enough recycling facilities." (Student 8's answer during the semi-structured interview)

Based on the responses above, it can be deduced that the students did not know the amount of trash and inadequacy of recycling facilities in Istanbul before reading the article given in the Trash Trouble MEA. In other words, they gained information about the trash issue, which is one of the environmental issues that we have, through the Trash Trouble MEA.

Similarly, the students learned information about excess amount of waste material, wastes taking up too much space, and lack of storage areas for the Minimum Waste, Maximum Pencil Box MEA. Below are some examples of the answers given by the students related to this theme.

> "I noticed that the amount of waste was very much even when making a pencil box." (Student 6's answer during group presentations)

> "We learned that there are many environmental problems such as excess waste." (Student 5's answer during group presentations)

> "We produce a lot of waste. I noticed that we must decrease the amount of waste we produce." (Student 14's answer during group presentations)

> "Thanks to the reading passage that we read before the problem, I learned that wastes take up too much space, and there is not enough space to store them." (Student 2's answer in the post-activity participant form)

These examples shows that the students realized that there are so much waste materials, too much space is needed to store them; however, there is not enough space for storage. This means that they gained knowledge about waste issue, which is one of the environmental issues, through the Minimum Waste, Maximum Pencil Box MEA.

4.3.2 Raising awareness about the issue: Thinking about action strategies

The students raise awareness about the issue stated that they wanted to inform the people around them and think of what can be done to reduce the amount of waste for the Trash Trouble MEA. Below are some examples of the answers given by the students related to this theme.

> "I informed my family that we should throw our garbage into the recycling bins." (Student 3's answer in the post-activity participant form)

> "I informed my family and my younger brother as to the amount of waste." (Student 14's answer in the post-activity participant form)
>
> "We should use products that generate less amount of trash." (Student 8's answer during the semi-structured interview)
>
> "If we put cans of recycling for oil and batteries at the beginning and the end of every street, there will be less amount of garbage." (Student 10's answer during group presentations and in the letter)
>
> "We should use our things economically and recycle waste materials." (Student 12's answer in the post-activity participant form)

As seen, the students not only understood the importance of the issue but also emphasized what kind of action strategies were necessary under favor of Trash Trouble MEA. For instance, they emphasized the usage of recycling, thinking about profit-damage balance, or informing their family to reduce the amount of waste. Those are beyond understanding the issue, and they particularly mentioned about the action strategies that could be done.

In a similar vein, the students raise awareness or inform the people around them, and think of what can be done to reduce the amount of thanks to the Minimum Waste, Maximum Pencil Box MEA. Below are some examples of the answers given by the students related to this theme.

> "My brother wastes a lot of blank papers while cutting out his drawings on the paper. Thus, I informed him about this issue." (Student 12's answer in the post-activity participant form)
>
> "We can use products that will reduce the amount of waste." (Student 8's answer during the semi-structured interview)

> “We can both make profit and reduce our damage to the environment by producing more products with less waste.” (Student 10’s answer during group presentations)

> “We should focus more on recycling. Therefore, we can use cans of recycling.” (Student 14’s answer during the semi-structured interview)

As it is seen in these responses, the students not only understood the importance of the issue but also emphasized what kind of actions were necessary with the help of the Minimum Waste, Maximum Pencil Box MEA. To illustrate, usage of recycling, thinking about the profit-damage balance, or self-control about consumption of paper could be some action strategies to reduce the amount of waste. Those actions often involved the ones that they could individually do in their daily lives.

CHAPTER 5

CONCLUSION AND DISCUSSION

The purpose of this study was to examine 7th grade students' learning residuals about mathematics and environmental issues in model-eliciting activities that were designed to address an environmental issue - waste management. In this chapter, conclusions that were explained in detail in the previous chapter are summarized and discussed. In addition, implications of the study and recommendations for further research are presented based on the conclusions of this study.

5.1 Discussion of the Findings

The discussion of the findings are presented under two sections based on the research questions. These sections are mathematical learning residuals and environmental learning residuals, respectively.

5.1.1 Mathematical Learning Residuals

Based on the findings related to mathematical learning residuals, the students used algebra, pattern and generalization, ratio and proportion, and arithmetic average concepts in the Trash Trouble MEA. They used geometry (circle, right circular cylinder, rectangle and square), ratio and proportion, area and perimeter concepts in the Minimum Waste, Maximum Pencil Box MEA.

The findings of the students' solutions in the Trash Trouble MEA indicate that all of the fourteen 7th grade students in five groups solved the first part of the problem. On the other hand, while the 1st, 3rd, 4th and 5th groups developed a mathematical model, the students in the 2nd group solved the first part of the problem without developing a model. The solutions of 1st, 3rd, 4th and 5th groups included mathematical models since they used a model of algebraic expression or a ratio. The solutions of the students in the 2nd group did not include a model since they only made a guess without any computation. In a similar vein, for the second part of the problem, while the 1st and 3rd groups developed models using a ratio, the 2nd and 5th groups did not solve the problem or develop a model since they did not do any work. In addition, the students in the 4th group did not develop a model since they did not find what was expected in the question and did not make any revisions.

The findings of the students' solutions in the Minimum Waste, Maximum Pencil Box MEA reveals that all groups except the 2nd one solved the problem and developed a mathematical model in the first part of the problem. The 1st, 3rd and 5th groups developed a visual model by placing the rectangles and circles based on geometric reasoning while the 4th group developed a model of expression by dividing the area of the layer by the total area of the materials. The solution of students in the 2nd group did not include a model since they placed the rectangles and circles randomly without making sense. Similar to the findings of the first part, the students in the 2nd group did not solve the second part of the problem. The group's productions in these two MEAs were summarized in Table 5.1 below.

Table 5.1 The summary of the groups' model productions

Groups	Trash Trouble MEA		Minimum Waste, Maximum Pencil Box MEA	
	Part 1	**Part 2**	**Part 1**	**Part 2**
Group 1	*	*	*	+
Group 2	+	-	-	-
Group 3	*	*	*	+
Group 4	*	-	*	+
Group 5	*	-	*	+

* : Groups that solved the problem with a model

+ : Groups that solved the problem without a model

- : Groups that could not solve the problem

As can be seen from the table above, the performance of the 1st and 3rd groups was better than that of other groups since they solved the first and second parts of the Trash Trouble MEA by developing models. They solved the Minimum Waste, Maximum Pencil Box MEA by developing partial models – by using models in the first part of the problem. The 2nd group could not develop any model for either MEAs, and they could not solve the Minimum Waste, Maximum Pencil Box MEA completely. The 4th and 5th groups solved the first part of the Trash Trouble MEA by developing models, but they could not solve the second part of the problem. Similar to the 1st and 3rd groups, the 4th and 5th groups solved the Minimum Waste, Maximum Pencil Box MEA by developing partial models.

Based on the findings of this study, there were five main conclusions. Firstly, the current study found that most of the five groups developed similar ideas and used similar mathematical concepts in both questions. For example, in the Trash Trouble MEA, most of the five groups tried to find the amount of increase/decrease in the amount of trash between years using numbers and operations. In the Minimum Waste, Maximum Pencil Box MEA, the 1st, 4th and 5th groups used ratio and proportion - by using a smaller layer - to find the number of pencil boxes they could place on the layer. Moreover, the 1st, 2nd, 3rd and 5th groups used the right circular cylinder and its properties while solving the Minimum Waste, Maximum Pencil Box MEA. This finding seems to be consistent with other researchers that found that two groups of 7th grade students showed similar mathematical ideas and cyclical processes while solving the modeling problem called Weather Problem (İnan Tutkun & Didiş Kabar, 2018).

The second conclusion was that although some groups did not generate a model or find a solution for the problems, some groups developed/used powerful mathematical ideas and/or a powerful model with multiple mathematical knowledge. Similar to this conclusion, in the study of Mousoulides and English (2011), two out of six groups did not develop suitable models while the other four groups developed suitable models with various mathematical concepts such as average or equation. In this study, the students in the 3rd group developed models using arithmetic average and build-up strategy (between ratio), and the students in the 1st group developed models using algebra, pattern and generalization, arithmetic average, and ratio and proportion for the Trash Trouble MEA. In the first part of the solution found by the students in the 3rd group, arithmetic average was useful since they found the average amount of trash per year. In the first part of the solution found by the students in the 1st group, algebraic pattern was useful since they found the amount of trash for the targeted year based on this pattern.

For the Minimum Waste, Maximum Pencil Box MEA, the students in the 1st group developed a geometrical model using ratio and proportion, right circular cylinder, rectangle, circle and square concepts. Right circular cylinder, rectangle, circle and square concepts were necessary to solve the Minimum Waste, Maximum Pencil Box MEA since metal pencil box is a kind of right circular cylinder. Students had to know the materials required to make a pencil box which is the net of a right circular cylinder consisting of a rectangle and two circles. Mathematical ideas involved in models of other groups are summarized in Table 5.2 below.

Table 5.2 Mathematical concepts used in the groups' models

Groups	**Mathematical Concepts used in the Groups' Models**	
	Trash Trouble MEA	**Minimum Waste, Maximum Pencil Box MEA**
Group 1	algebra pattern and generalization arithmetic average ratio and proportion	ratio and proportion right circular cylinder rectangle square circle
Group 2	-	right circular cylinder rectangle square circle
Group 3	arithmetic average build-up strategy (between ratio)	right circular cylinder rectangle square circle
Group 4	unit ratio	ratio and proportion area
Group 5	algebra pattern and generalization	ratio and proportion right circular cylinder rectangle square circle

This finding is in accord with recent studies in terms of including important mathematical topics in the models (Aliprantis & Carmona, 2003; Mousoulides et al., 2009; Stohlman, 2017). In Stohlman's study (2017), five groups that included 19 middle school students developed productive models including measurement (length and height), algebraic equation and ratio in the MEA called Bigfoot. Likewise, Aliprantis and Carmona (2003) found that twelve groups of middle school students expressed their mathematical ideas and developed models using a representational systems in the modeling activity - an adapted economics problem. In addition, Mousoulides et al. (2009) reported that twenty-two 11-year-old students constructed various models such as algebraic or graphical while solving the complex environmental problem of water shortage.

Thirdly, as can be seen from the density of the mathematical concepts in the table above, there was an obvious difference in the solutions of the first and second problems. The reason for this difference might be that the Minimum Waste, Maximum Pencil Box MEA involved more geometrical and visual elements compared to the Trash Trouble MEA because of the content of the problem. Furthermore, there were differences between the models generated by the students in the first and second problems. This means that the students generally developed verbal and algebraic models in the Trash Trouble MEA. On the other hand, they generally developed visual models in the Minimum Waste, Maximum Pencil Box MEA. A possible explanation for this difference might be that the models in the second problem had more geometric aspects due to the content of the problem. This conclusion is compatible with the study of Hıdıroğlu and Özkan Hıdıroğlu (2017). They found that the content of the problem was one of the problem-based factors that affected the students' models. In their study, 6th grade students constructed models containing more pictures and figures in the problems that asked for the height of straw bales and the maximum number of vehicles that could be parked in the garden in front of the school building. On the other hand, they constructed models containing more tables and lists in the problems that asked for an approximate number of students in the school and the average amount of water consumption in a week.

The fourth conclusion was that in both problems, the models of some groups were stronger while other models were weaker from the researcher's perspective. In the Minimum Waste, Maximum Pencil Box MEA, the model of the 1st group was strong since they placed three circles into a rectangle to make a more organized placement with less amount of waste. In the Trash Trouble MEA, the model of the 2nd group was weaker since they concluded the problem by making a guess without any mathematical computation or reasoning. The students' mathematical background or their prior knowledge might be possible reasons for this conclusion. For instance, the students in the 4th group had some misconceptions related to area and perimeter concepts in the Minimum Waste, Maximum Pencil Box MEA. This conclusion is in agreement with Hıdıroğlu and Özkan Hıdıroğlu's (2017) findings which showed that some students tried to conclude problems with less complicated models due to their prior mathematical knowledge. On the other hand, in their study, some students did not develop a suitable model because they confused the units of length and area concepts in the modeling problem that asked for the height of straw bales. The second possible reason might be time-management. Since mathematical modeling is a time-demanding process, while the time allocated for modeling problems in this study was sufficient for some groups to create strong models, it may have been insufficient for others. This finding further supports the idea of Dedebaş (2017) who stated that time limitation led to development of less strong models for 5th grade students.

The last conclusion emerging from this study was that the students in the 2nd group did not generate a model for either problems (see Table 5.1 above). Similar to this finding, Aliprantis and Carmona (2003) stated that while some groups generated models, others could not generate a model. There might be some reasons for that conclusion. Firstly, according to Blum and Ferri (2009), the first step of the modeling cycle is the situation model which means that problem solvers should understand the problem situation. In this study, the students in the 2nd group expressed that they did not understand the problem. Thus, they could not generate a model. This finding corroborates the ideas of Jankvist and Niss (2020) who found that majority of 315 Danish students (grades 10, 11 and 12) had difficulty in understanding problems while working with six modeling tasks. This finding is also supported by Hıdıroğlu and Özkan Hıdıroğlu (2017), who concluded that understanding the problem was one of the student-based factors that affected students' models. In their study, the students who could not understand the problem either could not generate a model or drew irrelevant picture-types models. Secondly, they could not generate a model because of time management since modeling is a time-demanding process; even though they develop an initial idea, constructing a model requires a certain amount of time. This finding supports the evidence from the study of Deniz and Kurt (2022) who found that students had problems with time management while developing models. This finding is also consistent with the findings of Greefrath's (2013) study. He found that secondary school students who spent necessary time to understand the problem and plan the process used mathematical terms and models while solving problems. However, the students who did not spend the necessary time to understand the problem and plan the process did not use mathematical terms and models sufficiently.

5.1.2 Environmental Learning Residuals

In schools, environmental education is necessary to protect and improve environment (Loughland et al., 2010). Rickinson et al. (2009) stated that two aims of environmental learning are to raise awareness and to take actions as to the environment by developing students' critical thinking. Based on the findings of this study related to environmental learning residuals, the students raised awareness for two things: (1) to understand the current and local situation (that calls a risk) and (2) to develop action strategies for a sustainable future. More specifically, the students were informed about the trash issue in Istanbul which is related to understanding the current/local situation, and they thought of possible action strategies to solve this issue through the Trash Trouble MEA. This awareness came from the content of the modeling problem, and the mathematical models that the students developed as a solution for these environmental issues were an important aspect of their modeling process.

In the first part of the Trash Trouble MEA, which asked for how much trash will be produced in Istanbul in 2025 have raised awareness about a local situation since the question included information about the amount of trash in Istanbul between 2004 and 2019. The students have realized the seriousness of the local situation by examining the data indicating that the amount of trash increases almost every year. It was also evident in Student 1's statement from the post-activity participant form as given in the findings in detail: "I was aware that there is too much trash in Istanbul, and that even one person can produce thousands of trash." Moreover, the context of the problems should be authentic or taken from the real-life to make learning permanent, and model-eliciting activities are simulations of real-life (Lesh et al., 2000; Van de Walle et al., 2013). In this study, students' awareness related to local waste management increased since the Trash Trouble MEA focused on the trash issue where students lived. In other words, the context of the modeling problem that was from students' immediate environment helped them to realize the local issue. The 1st group developed a model with an algebraic pattern to present that the amount of trash increases continuously. In other words, the students raised awareness about the amount of trash in Istanbul and reflected this awareness in the model they developed. In addition, at the end of the Trash Trouble MEA, there was a statement: "You can also include suggestions in your letter as to how the amount of trash produced each year can be reduced." This statement led students to develop action strategies for a sustainable future. It was also evident in Student 10's statement from the group presentations as given in the findings in detail: "If we put recycling bins for oil and batteries at the beginning and the end of every street, there will be less amount of garbage."

In the Minimum Waste, Maximum Pencil Box MEA, the first part of the problem was related to developing a model to minimize the amount of waste materials when making the bottoms and sidewalls of a metal pencil box on a 100 cm x 100 cm square layer. This part has raised awareness about the current/local situation because it took students' attention to the wasted material and set the goal of minimizing the amount of waste materials while placing the bottoms and sidewalls of the metal pencil box on the layer. It was also evident in Student 6's statement from the group presentations as given in the findings in detail: "I noticed that there was so much waste even when making pencil boxes." In the solution of the 1st group, they developed a model by placing three circles into a rectangle instead of placing rectangles and circles randomly to provide minimum waste with this awareness. Moreover, the second part of the problem was, "Assuming that the pencil box's radius and height will not change, what should be the size of the square layer to be used to have minimum amount of waste and the maximum number of pencil boxes?" This part led students to develop action strategies for a sustainable future by associating the problem with other real-life situations. In the solution of the 5th group, the students did not change the sizes of the square layer since they thought that there was less amount of waste. It means that they understood the seriousness of the risk, and they did not want to increase the amount of waste by changing the sizes of the square layer. Apart from the questions in both problems, the reading passages and readiness questions raised awareness about the current situation and led students to develop action strategies.

In short, it can be said that the content of the modeling problems and the students' solutions/models for these problems led students to understand current and/or local situations and propose action strategies. This conclusion is also parallel to the E-STEM education which allows students to realize environmental problems during the problem-solving process, raise their awareness and take actions so as to solve these problems (NAAEE, 2013). In other words, the students noticed the environmental issue and tried to find a solution to this issue while engaging in model-eliciting activities in this study. This finding further supports the idea of Gürbüz and Çalık (2021) who found that seventh grade students' awareness related to waste management increased, and they thought they were responsible for their immediate environment through interdisciplinary mathematical modeling activities. Therefore, mathematical learning residuals and environmental learning residuals are integrated. This integration is also related to STEM education and the interdisciplinary nature of mathematical modeling. It is related to STEM education since students join a problem-solving process by combining two or more STEM fields which are science, technology, engineering and mathematics (Sanders, 2009; Shaughnessy, 2013). It is also related to the interdisciplinary nature of mathematical modeling since rather than traditional education that does not include real-life situations, modeling includes interdisciplinary and authentic real-life problems in nature (Erbaş et al., 2014). Therefore, the model-eliciting activities used in this study associated mathematics with science (an environmental problem of waste/trash). Similarly, in the study of English and Mousoulides (2011), an environmental engineering problem (related to developing a model to supply water for Cyprus) was presented to thirty-eight 11-year-old students in Cyprus. In their study, some of the groups associated mathematics with science (water storage, sea pollution or energy consumption) while developing their models by analyzing data.

Doğan et al. (2019) stated that students might understand mathematical concepts in real-life by integrating mathematics and other disciplines through interdisciplinary mathematical modeling activities centered on STEM education. In this study, interdisciplinary model-eliciting activities that address STEM education enabled students to solve the problems by using mathematical concepts and science (environmental issues). This finding was in agreement with the findings of the study conducted by Mousoulides and English (2011). They concluded that twenty 12-years-old students used mathematical and science concepts while solving the engineering modeling problem related to natural gas resources and consumption. In another study by English and Mousoulides (2015), 48 twelve-years-old students associated science, engineering and mathematics with each other and used them while working on an interdisciplinary mathematical modeling problem that was related to developing a model to construct a new bridge.

In this study, the students gained awareness related to one of the environmental issues of trash/waste. In other words, the students gained knowledge about the environmental issue that they did not know before. This finding is consistent with the findings of Makki et al. (2003) in such a way that Labanese secondary school students were lack of knowledge related to environmental issues such as recycling or pollution. This awareness led students to propose action strategies about what they can do to solve these issues. It was also evident in Student 8's statement from the semi-structured interview as given in the findings in detail: "We can use products that will reduce the amount of waste." In addition, this awareness may lead them to make it a manner of life to protect the environment. This finding reflects those of Arı and Yılmaz (2017) who found that middle school students developed pro-environmental behaviors such as using eco-friendly products with the help of environmental awareness. This finding is also supported by the findings of the study conducted by Susilawati et al. (2017). They found that seventh-grade students improved their attitudes towards the environmental issue of waste and its management at both school and home by using project-based learning with mind maps.

5.2 Implications of the Study

There were two major implications for educational practices in the light of the findings of this study. In this study, two model-eliciting activities that included environmental issues were implemented to 7th grade students. Although all of the groups did not develop a model or reached a solution, most of them used various mathematical contents, showed mathematical ideas and generated strong models. Considering all of these findings, it can be said that teachers are suggested to use model-eliciting activities more in their lessons. On the other hand, these findings may help mathematics teachers understand possible mathematical and environmental learning residuals of middle school students in these MEAs. Therefore, middle school mathematics teachers can use these problems directly, adapt problems based on their lesson plans, or they can write model-eliciting activities based on these problems. In addition, the students developed different kinds of models while working on model-eliciting activities that included environmental issues in this study. These activities can create a rich mathematical discussion environment. Hence, mathematics and science teachers can develop this kind of realistic, interdisciplinary and purposeful problems together. These findings may also help mathematics teacher educators organize pre-service teacher training programs so that pre-service mathematics teachers gain awareness about the mathematical and environmental learning residuals of middle school students.

Moreover, this study demonstrates that the students developed solutions and strong models using multiple mathematical knowledge in model-eliciting activities, gained insights into an environmental problem and thought about actions for future. Thus, curriculum developers and textbook writers may include environmental-based MEAs in mathematics curriculum and textbooks to raise students' awareness of environmental issues and enable them to find solutions for real-life situations.

5.3 Recommendations for Further Studies

Based on the findings of this study, some recommendations can be given for further studies. First of all, this study was limited to 14 seventh grade middle school students and conducted during distance education because of Covid-19 pandemic. In order to extend the findings and eliminate the limitations of this study, collection of an extended samples with the same grade level or other grade levels and carrying out a similar research in face-to-face setting can be a strand of future research. Moreover, a longitudinal study can be conducted to examine middle school students’ development of mathematical and environmental learning residuals in model-eliciting activities instead of collecting data at once.

Secondly, there were two model-eliciting activities related to the issue of trash/waste as an environmental problem in this study. In future investigations, it might be possible to integrate different environmental problems with model-eliciting activities to draw attention to and take action for a sustainable future. For instance, model-eliciting activities may be developed related to climate change, which is a serious environmental problem. Furthermore, an interdisciplinary study can be conducted with other STEM area(s) such as technology instead of only science. To illustrate, the technology aspect can be added to the strand of this study. To do this, students can use mathematical software such as GeoGebra while solving problems.

Thirdly, this study focused on 7^{th} grade students’ learning residuals about mathematics and environmental issues in model-eliciting activities that address environmental issues. In addition to learning residuals related to mathematics and environmental issues, students gain an understanding of how mathematics is beneficial in real-life situations. This understanding can enable students to develop positive attitudes towards mathematics. Therefore, further research may focus on the attitudes or beliefs of middle school students who engage in model-eliciting activities regarding mathematical modeling and environmental issues.

Lastly, model-eliciting activities used in this study serves as an initiating experience for students to understand current and/or local situations and to develop action strategies for that situation. Therefore, these kind of activities may be implemented in the eco-schools by integrating science and mathematics so that students gain awareness related to local situations and propose what can be done to solve the local problem.

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APPENDICES

A. Model-Eliciting Activity 1

İstanbul'da bir kişi yılda 287 kg çöp üretiyor, yıllık çöp miktarı 6 milyon tona yaklaşıyor

22.01.2020 – Hazar Dost - https://medyascope.tv

İstanbul Büyükşehir Belediyesi'nin (İBB) *Açık Veri Portalı*, 2004-2019 yılları arasındaki evsel atık verilerini yayımladı. Bu verilere göre 2019'da İstanbul'da toplam 8 milyon 827 bin ton çöp üretildi. İstanbul, 2004 yılında 3 milyon 216 bin ton çöp üretiyordu.

Verilere göre, İstanbul'da yaşayan bir kişi yılda 287 kilogram çöp üretiyor. Ortalama üç kişilik bir ailenin evinden günlük 2,5 kilograma yakın evsel atık çıkıyor.

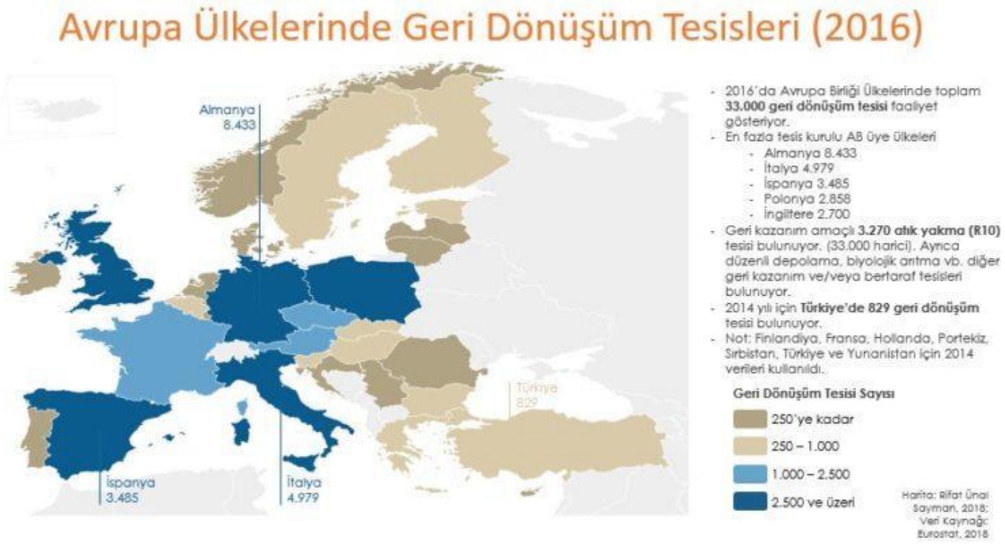
En fazla çöpü Esenyurt ilçesi üretiyor

Evsel atık üretiminin en fazla olduğu ilçe Esenyurt. 2004-2019 yılları arasında 2 milyon 530 ton çöp üreten Esenyurt, 2019 yılında ise 262 bin 284 tonluk çöp üretimiyle ilk sırada yer alıyor.

Türkiye'de 829 geri dönüşüm tesisi var

İBB'nin geri dönüşüm programı doğrultusunda İstanbul'un çöplerinden elektrik enerjisi de üretiliyor. İstanbul'da 800 bin kişinin elektrik enerjisi ihtiyacı evsel atıkların geri dönüştürülmesiyle karşılanabiliyor. Ancak geri dönüşüm tesislerinin sayısı yine de Avrupa ülkelerinin gerisinde.

Türkiye, Avrupa'da Fransa'yla beraber en fazla evsel atık üreten ikinci ülke. Fakat atıkların geri dönüştürülmesi konusunda Avrupa'nın gerisinde. Almanya'da 8 bin 433, İtalya'da 4 bin 979, İspanya'da ise 3 bin 485 geri dönüşüm tesisi bulunuyor. Türkiye'de ise geri dönüşüm tesisi sayısı 829.



Bölgesel Çevre Merkezi (REC Türkiye) Direktörü Rıfat Ünal Sayman'ın araştırması da, evsel atık üretiminin önümüzdeki yıllarda ciddi şekilde artacağını ortaya koyuyor.

Türkiye'de 29 il çöplerin yüzde 99'unu geri dönüştüremiyor

Aralarında iki büyükşehir belediyesinin de olduğu 29 il, günlük atıkların yüzde 99'unu geri dönüştüremiyor. Atıkların dönüştürülmesi konusunda yüzde 53'le Ankara birinci sırada. İstanbul ise çöplerin sadece yüzde 9'unu dönüştürebiliyor.

HAZIRLIK SORULARI

1. İstanbul Büyükşehir Belediyesi'nin (İBB) Açık Veri Portalı verilerine göre, evsel atık üretiminin en fazla olduğu ilçe hangisidir ve 2019 yılında kaç ton çöp üretmiştir?

2. Avrupa ülkeleri arasında en fazla evsel atık üretimi yapan ülke hangisidir?

3. İstanbul'da kaç kişinin elektrik enerjisi ihtiyacı evsel atıkların geri dönüştürülmesiyle karşılanabiliyor?

4. TÜİK 2018 verilerine göre belirlenen evsel atık geri kazanımındaki ilk üç ili kazanım oranlarıyla birlikte yazınız.

ÇÖP SORUNU

İstanbul Büyükşehir Belediyesi, İstanbul'da üretilen çöp miktarının her geçen gün artmasından dolayı geleceğimiz adına endişe etmektedir. İstanbul'da toplanan evsel atıklar, İstanbul'un her iki yakasında bulunan düzenli depolama sahalarında 20 yılı aşkın süredir kesintisiz olarak toprağı, suyu ve havayı kirletmeyecek şekilde güvenle ortadan kaldırılmaktadır. Ancak bu sahalar dolmak üzere olduğu için kapanma tehlikesi ile karşı karşıyadır. Bu sebeplerle, Belediye Başkanı Ekrem İmamoğlu ve çalışma arkadaşları çöplerin nerede saklanacağı ve nasıl yeniden kullanılabileceği konusunda çözümler bulunması gerektiğini düşünmektedir.

Problem Durumu:

- İstanbul'daki çöple ne yapılacağına yönelik plan yapmak adına, gelecekte İstanbul'da ne kadar çöp üretileceği konusunda bir tahmine sahip olmak önemlidir. Bu sebeple, İstanbul Büyükşehir Belediye Başkanı Ekrem İmamoğlu ve çalışma arkadaşları, İstanbul'un 2025 yılında ne kadar çöp üreteceğini belirlemede sizden yardım istiyor. 2025 yılında üretilecek çöp miktarını belirlemenize yardımcı olması için Çöp Sorunu Problemi-Veriler başlıklı sayfadaki verileri kullanınız. Ekrem İmamoğlu'na, İstanbul'un 2025 yılında üreteceği çöp miktarının ne olacağına dair hesaplamalarınızın yer aldığı ve hesaplamanızı nasıl yaptığınıza ilişkin prosedürünüzü açıklayan bir mektup yazınız. Prosedürünüz İstanbul'daki çöp üretimi ile ilgili yeni verilerin mevcut olacağı gelecek yıllarda kullanılabilecek şekilde açıklanmalıdır.
- 2025 yılında çöp gazından 650.000 MWh elektrik enerjisi üretilebilmesi için atık miktarı ne kadar olmalıdır?
- Türkiye'de günlük kişi başına toplanan ortalama atık miktarı 1,17 kg. Bu miktar İstanbul'da 1,30 kg, Ankara'da ise 1,14 kg'dır. Uzmanlar İstanbul'da herhangi bir zamanda üretilecek atık miktarının nüfusla doğru orantılı olarak Ankara'daki atık miktarından fazla olacağını düşündüğüne göre, İstanbul'daki atık miktarının Türkiye ortalamasına düşmesi için bir kişi yıllık ortalama kaç kg atık üretmelidir?
- Ayrıca, mektubunuza her yıl üretilen çöp miktarının nasıl azaltılabileceğine dair önerilerinizi de ekleyebilirsiniz.

ÇÖP SORUNU PROBLEMİ – VERİLER

Tablo 1. Yıllara göre İstanbul'da üretilen atık miktarı

Yıllar	Atık Miktarı (ton)
2004	3.216.787
2006	3.321.910
2008	3.267.190
2010	3.372.096
2012	3.580.645
2014	3.888.079
2015	4.288.187
2016	4.805.188
2017	5.414.332
2018	5.930.460
2019	5.927.702

Tablo 2. Yıllara göre İstanbul'da atıktan elde edilen geri kazanım miktarları

Geri Kazanım Verisi	**2004**	**2009**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**
Geri Dönüşebilir Malzeme Miktarı (ton)	1.513	8.454	17.425	7.069	18.815	10.974	9.163	8.832
Atıktan Üretilmiş Yakıt Miktarı (ton)	-	1.087	35.552	63.894	39.602	13.291	21.757	26.417
Çöp Gazından Üretilen Elektrik Enerjisi Miktarları (MWh)	5.938	70.895	336.547	358.125	404.330	450.690	499.312	500.278

*Veriler İstanbul Büyükşehir Belediyesi (İBB) Açık Veri Portalı sayfasından (https://data.ibb.gov.tr/) alınıp düzenlenmiştir.

B. Model-Eliciting Activity 2

Atıkları Azaltmak Neden Bu Kadar Önemli?

21 Kasım 2019 – Solo - https://www.solo.com.au/latest_news/why-its-so-important-to-reducewaste/

Atık söz konusu olduğunda, çoğumuz geri dönüştürülebilir ürünlerimizi genel atıklardan ayırmanın ve çöplerimizi doğru çöp kutusuna koymanın temellerini biliyoruz. Ancak, daha azımız atıkların nereye gittiğini ve çevre üzerindeki etkisini düşünüyoruz.

Geri dönüşüm muhtemelen çevreye duyarlı olduğunuzu hissetmenin en kolay yoludur ve kesinlikle faydalıdır, ancak atıkların azaltılmasının gelecek nesiller için sürdürülebilir bir gelecek yaratmada eşit derecede önemli olduğunu biliyor muydunuz?

Düzenli depolama alanlarının tükenmesine ve her yıl düzenli depolama alanlarına 6,2 milyon tondan fazla organik atık göndermeye devam ettiğimiz için Avustralya şu anda potansiyel bir atık kriziyle karşı karşıyadır. Bu nedenle geri dönüşüm kesinlikle teşvik edilmeli, ancak yeniden kullanmak ve atık miktarını azaltmak gibi diğer seçeneklere de bakılmalıdır.

Atıkları azaltmak, sabah kahveniz için kullan-at bir bardak yerine tekrar tekrar kullanılabilen fincan kullanmak veya şişelenmiş su satın almaktan kaçınmak kadar basit olabilir. Daha iyi atık yönetimine doğru büyük bir hamle gibi görünmeyebilir, ancak herkes ne kadar atık ürettiğinin farkına varırsa topluca gezegenimiz ve geleceği üzerine olumlu bir etki etmeye başlayabiliriz.

Atık miktarını azaltmanın temel nedenleri:

- Atıkların geri dönüşümü ve azaltılmasının önemli olmasının birçok nedeni vardır. Çevresel nedenler genellikle çok konuşulsa da atık miktarını azaltmak, ürünleri geri dönüştürmek için daha fazla iş imkanı yaratır. Bu durum finansmanımız üzerindeki olumlu bir etkiye aynı zamanda olumlu bir sosyal etkiye sahiptir. Yalnızca gerçekten ihtiyacınız olanı satın alarak, ürünleri yeniden kullanarak uygun atık bilinci ile paradan tasarruf edebilirsiniz.
- Yeni malzemeler oluşturmaya karşı, var olan malzemeleri geri dönüştürmek için daha az enerji kullanılır. Bu nedenle gerekli yeni kaynakların miktarını sınırlandırarak büyük miktarda enerji tasarrufu sağlanabilir.
- İsrafımızı azaltarak kaynaklarımızı da koruyoruz. Alüminyum, petrol ve ağaçlar gibi kaynakların tümü teneke kutular, plastik torbalar ve kağıt ambalaj gibi yeni malzemeler yapmak için kullanılır.
- İsrafı azaltmanın en büyük nedenlerinden biri de depolama alanlarımızdaki alanı korumak ve önemli derecede alan kaplayıp hava/su kirliliği kaynağı olan daha fazla depolama alanı inşa etme ihtiyacını azaltmaktır.

Madencilik, rafine etme ve üretim süreci, çevreye zarar veren tehlikeli sera gazı emisyonlarının yayılmasından sorumludur. Elimizdeki atık miktarını geri dönüştürerek, yeniden kullanarak ve azaltarak, çocuklarımız ve torunlarımız için daha sürdürülebilir bir gelecek inşa etmeye yardımcı oluruz.

HAZIRLIK SORULARI

1. Atık miktarını azaltmanın üç temel nedenini yazınız.

2. Atık miktarını azaltmak için bireysel olarak alınabilecek önlemlere ne gibi örnekler verebilirsiniz?

3. Yeni malzemeler oluşturmaya karşı, var olan malzemeleri geri dönüştürmek neden önemlidir?

4. Atıkların depolanma alanlarında ne gibi sorunlar yaşanabilir?

MİNİMUM ATIK, MAKSİMUM KALEMLİK!

KİME: Mühendislik Ekibi

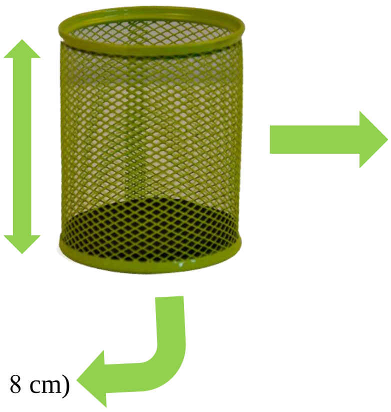
KİMDEN: İstanbul Mimarlık ve Mühendislik Şirketi, CEO: İmalat Malzemeleri

KONU: Metal Kalemlik Üretimi

ABC Kırtasiye, metal kalemlik üretimi konusunda şirketimizle iletişime geçti. Bu daha önce üretmediğimiz yeni bir ürün olacaktır, bu nedenle üretim için etkin bir prosedürümüz olduğundan emin olmamız önemlidir. Şirket bizden boşa giden malzeme miktarını en aza indirecek şekilde istenilen ölçülerde renkli metal kalemlikler üretmemizi istiyor.

Metal kalemlikleri üretmek için tam otomatik bir makine satın aldık. Makine gerekli tüm malzemeleri tek bir tabakadan tek seferde çıkaracaktır. Bu nedenle de, bu tabakanın üzerine gerekli şekillerin gerekli ölçülerde ve gerekli sayıda dizgisinin yapılması gerekmektedir. Makine kalemliği oluşturmak için gerekli iki şekli kestiğinde şirketimizin boşa giden malzemeyi en aza indirgemek için tabakaya en iyi dizginin nasıl olacağını bulması önemlidir.

kalemlik boyu : 10 cm



kalemlik duvarı

kalemlik tabanı (çap: 8 cm)

Bir raporda lütfen aşağıdaki bilgilerle cevap verin:

- Ekibinizin 100 cm x 100 cm’ lik bir metal malzeme tabakasına verilen boyutlardaki eşdeğer sayıda kalemlik tabanını ve kalemlik duvarını yerleştirecek şekilde bir yöntem açıklaması gerekmektedir. (Bu yerleşimi yaparken kullanacağınız yöntemin atık malzemeyi en aza indirgeyecek şekilde olmasına dikkat ediniz.)
- Kalemlik boyunun ve yarıçapının değişmeyeceğini varsayarak, minimum miktarda atık ve maksimum sayıda kalemlik için kullanılacak kare tabakanın ölçüleri ne olmalıdır?
- Yapacağınız işlemlerde π’yi 3 alınız.

İşbirliğiniz için şimdiden teşekkür ederiz.

İstanbul Mimarlık ve Mühendislik Şirketi, CEO

C. The Post-Activity Participant Form

ETKİNLİK SONRASI KATILIMCI FORMU

Ad/Soyad: **Tarih:**

1. Bu problemi çözerken hangi matematiksel konuyu/kavramı/beceriyi kullandınız?

2. Kullandığınız matematiksel konu/kavram/beceriyi ne kadar anladınız?

( ) Çok Kötü ( ) Kötü ( ) Orta ( ) İyi ( ) Çok iyi

Seçim nedeninizi açıklayınız:

3. Bu problemin çevresel sorunlara karşı farkındalık kazanmanızda bir etkisi oldu mu? Nasıl?

4. Bu problemin disiplinler arası (branşlar arası) olması size ne gibi bir katkı sağladı?

D. METU Human Subjects Ethics Committee Approval

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

FEN BİLİMLERİ ENSTİTÜSÜ MÜDÜRLÜĞÜ
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T:+90 312 210 22 92
F:+90 312 210 79 59
wwwfbe@metu.edu.tr
www.fbe.metu.edu.tr

Sayı : 44280379-000-E.36804
Konu : Öğrenci Gamze Baktemur- Anket Çalışması

Rektörlük Makamı

İlgi : Matematik Ve Fen Bilimleri Eğitimi Bölüm Başkanlığının 10.02.2021 tarihli ve 24422252-000-E.36614 sayılı yazısı

Matematik ve Fen Bilimleri Eğitimi EABD yüksek lisans programı öğrencisi Gamze Baktemur'un "STEM Eğitimine Hizmet Eden ve Çevresel Sorunlara Değinen Modelleme Etkinliklerinin Öğrenme Kalıntıları" başlıklı tez çalışması kapsamında, 26 Nisan-14 Mayıs 2021 tarihleri arasında, İstanbul- Sançaktepe ilçesi İl Milli Eğitim Bakanlığına bağlı Samandıra Ortaokulunda eğitim gören 20 yedinci sınıf öğrencisi ile çalışma yapması planlanmaktadır. Bu çalışma, ilgili Ana Bilimdalı Başkanlığı'nın görüşü ve İnsan Araştırmaları Etik Kurulu'ndan alınan 256 ODTÜ 2020 protokol numaralı Etik Kurulu tarafından onaylanmış ve Enstitümüzce uygun görülmüştür.

Gereği için bilgilerinize arz ederim.

Saygılarımla.

e-imzalıdır
Prof. Dr. Ayşegül ASKAN GÜNDOĞAN
Enstitü Müdür Yardımcısı

EK: Dağıtım Listesi (1 Sayfa)

OLUR

Bu belge 5070 sayılı Elektronik İmza Kanununa göre güvenli elektronik imza ile imzalanmıştır.
Belge Doğrulama Kodu : https://ebys.metu.edu.tr/verifyrecord/bg.aspx?id=534902C4-07F6-4AE1-B9E5-E07A05975DAF

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wwwfbe@metu.edu.tr
www.fbe.metu.edu.tr

e-imzalıdır
Prof. Dr. Tülin GENÇÖZ
Rektör Yardımcısı

Bu belge 5070 sayılı Elektronik İmza Kanununa göre güvenli elektronik imza ile imzalanmıştır.
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E. Permission Obtained from Ministry of National Education

T.C.
İSTANBUL VALİLİĞİ
İl Millî Eğitim Müdürlüğü

GÜNLÜDÜR
30.03.2021

Sayı : E-59090411-44-23331038
Konu : Anket ve Araştırma İzinleri

DAĞITIM YERLERİNE

İlgi : a) Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 21.02.2020 tarihli ve 2020/2 sayılı genelgesi.
b) Valilik Makamının 25.03.2021 tarihli ve 23012049 sayılı oluru.

Valilik Makamının Anket ve Araştırma İzinleri konulu ilgi oluru, anket izni uygun görülenlerin listesi, kullanılması uygun görülen ölçme araçlarının Müdürlüğümüzce mühürlenen örnekleri ekte gönderilmiştir.

Olur gereğince işlem yapılması, araştırma sonuç raporunun, araştırma bittikten sonra 2 (iki) hafta içerisinde Müdürlüğümüz Strateji Geliştirme Şubesine gönderilmesi hususlarında gereğini arz ederim.

Levent YAZICI
İl Millî Eğitim Müdürü

Ek:
1- Valilik Oluru (1 Sayfa)
2- Liste (1 Sayfa)
3- Ölçekler

Dağıtım:
Orta Doğu Teknik Üniversitesi Rektörlüğüne
İstanbul Sabahattin Zaim Üniversitesi Rektörlüğüne

Bu belge güvenli elektronik imza ile imzalanmıştır.
Adres : Binbirdirek Mah. İmran Öktem Cad. No: 1 Sultanahmet Fatih İstanbul
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E-posta : stratejigelistirme34@meb.gov.tr
Kep Adresi : meb@hs01.kep.tr
Belge Doğrulama : https://www.turkiye.gov.tr/meb-ebys
Bilgi İçin : Aydın BALTA
Unvanı : Veri Hazırlama ve Kontrol İşletmeni
İnternet Adresi : http://istanbul.meb.gov.tr/
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T.C.
İSTANBUL VALİLİĞİ
İl Millî Eğitim Müdürlüğü

Sayı : E-59090411-20-23012049
Konu : Anket ve Araştırma İzinleri

25/03/2021

VALİLİK MAKAMINA

İlgi : Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 21.01.2020 tarihli ve 2020/2 sayılı genelgesi.

Aşağıda bilgileri verilen araştırmaların; 6698 sayılı Kişisel Verilerin Korunması Kanununa aykırı olarak kişisel veri istenmemesi, öğrenci velilerinden açık rıza onayı alınması, yüz yüze eğitime geçmiş olan kurumlarımızda, Covid-19 tedbirlerinin araştırmacı ve ilgili kurum idarelerince alınması, bilimsel amaç dışında kullanılmaması, bir örneği Müdürlüğümüzde muhafaza edilen mühürlü ve imzalı veri toplama araçlarının kurumlarımıza araştırmacı tarafından ulaştırılarak uygulanması, katılımcıların gönüllülük esasına göre seçilmesi, araştırma sonuç raporunun kamuoyuyla paylaşılmaması ve araştırma bittikten sonra 2 (iki) hafta içerisinde Müdürlüğümüze gönderilmesi, okul idarelerinin denetim, gözetim ve sorumluluğunda, eğitim ve öğretimi aksatmayacak şekilde, 2020-2021 eğitim ve öğretim yılında ilgi genelge esasları dâhilinde uygulanması kaydıyla Müdürlüğümüzce uygun görülmektedir.

Makamınızca da uygun görüldüğü takdirde olurlarınıza arz ederim.

Levent YAZICI
İl Millî Eğitim Müdürü

OLUR
25/03/2021
Dr. Hasan Hüseyin CAN
Vali a.
Vali Yardımcısı

Ek:
1- Yazılar ve Ekleri (32 Sayfa)
2- Genelge (3 Sayfa)

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1/3
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Bilgi İçin : Aydın BALTA
Unvanı : Veri Hazırlama ve Kontrol İşletmeni
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T.C.
İSTANBUL VALİLİĞİ
İl Milli Eğitim Müdürlüğü

Sayı : E-59090411-20-23012049 25/03/2021
Konu : Anket ve Araştırma İzinleri

ANKET VE ARAŞTIRMA İZNİ UYGUN GÖRÜLENLER
ORTA DOĞU TEKNİK ÜNİVERSİTESİ

2020 - 2021 EĞİTİM VE ÖĞRETİM YILINDA GEÇERLİDİR

Araştırmacı	Yazı Tarihi	Sayısı	Araştırma Konusu	Araştırma Yeri	Araştırma Kişiler
Bora ALPAY	11.03.2021	289	Öğrencilerin Su Tüketimi Davranışlarının Değerler, İnançlar, Normlar ve Doğa ile Yakınlık Kapsamında İncelenmesi	Ataşehir ilçesi	Ortaokullarda öğrenim gören öğrencilere
Gamze BAKTEMUR	04.03.2021	282	STEM Eğitimine Hizmet Eden ve Çevresel Sorunlara Değinen Modelleme Etkinliklerinin Öğrenme Kalıntıları	Sancaktepe ilçesi	Ortaokul Öğrencilerine

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F. Parent Consent Form

VELİ ONAM FORMU

Sayın veli;

Çocuğunuzun katılacağı bu çalışma, "STEM Eğitimine Hizmet Eden ve Çevresel Sorunlara Değinen Modelleme Etkinliklerinin Öğrenme Kalıntıları " adıyla, 26.04.2021 – 14.05.2021 tarihleri arasında yapılacak bir araştırma uygulamasıdır.

Araştırmanın Hedefi: 7. sınıf öğrencilerinin STEM eğitimine hizmet eden ve çevresel sorunlara değinen modelleme etkinlikleri ile uğraşırken matematik ve çevresel sorunlarla ilgili neler öğrendiklerini incelemektir.

Araştırma Uygulaması: Anket (model oluşturma etkinlikleri), görüşme ve gözlem şeklindedir.

Araştırma T.C. Milli Eğitim Bakanlığı'nın ve okul yönetiminin de izni ile gerçekleşmektedir. Araştırma uygulamasına katılım tamamıyla gönüllülük esasına dayalı olmaktadır. Çocuğunuz çalışmaya katılıp katılmamakta özgürdür. Araştırma çocuğunuz için herhangi bir istenmeyen etki ya da risk taşımamaktadır. Çocuğunuzun katılımı **tamamen sizin isteğinize bağlıdır**, reddedebilir ya da herhangi bir aşamasında ayrılabilirsiniz. Araştırmaya katılmama veya araştırmadan ayrılma durumunda öğrencilerin akademik başarıları, okul ve öğretmenleriyle olan ilişkileri etkilenmeyecektir.

Çalışmada öğrencilerden kimlik belirleyici hiçbir bilgi istenmemektedir. Cevaplar tamamıyla gizli tutulacak, sadece araştırmacılar tarafından değerlendirilecek ve bilimsel amaçla kullanılacaktır.

Uygulamalar, genel olarak kişisel rahatsızlık verecek sorular ve durumlar içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden çocuğunuz kendisini rahatsız hissederse cevaplama işini yarıda bırakıp çıkmakta özgürdür. Bu durumda rahatsızlığın giderilmesi için gereken yardım sağlanacaktır. Çocuğunuz çalışmaya katıldıktan sonra istediği an vazgeçebilir. Böyle bir durumda veri toplama aracını uygulayan kişiye, çalışmayı tamamlamayacağını söylemesi yeterli olacaktır. Anket çalışmasına katılmamak ya da katıldıktan sonra vazgeçmek çocuğunuza hiçbir sorumluluk getirmeyecektir.

Onay vermeden önce sormak istediğiniz herhangi bir konu varsa sormaktan çekinmeyiniz. Çalışma bittikten sonra bizlere telefon veya e-posta ile ulaşarak soru sorabilir, sonuçlar hakkında bilgi isteyebilirsiniz.

Saygılarımızla,

Araştırmacı: Gamze BAKTEMUR
İletişim bilgileri: 0530 467 70 22 / gamzebaktemur@gmail.com

Velisi bulunduğum .... sınıfı .... numaralı öğrencisi... ……………………………………’in yukarıda açıklanan araştırmaya katılmasına izin veriyorum. (Lütfen formu imzaladıktan sonra çocuğunuzla okula geri gönderiniz*).

…./…../……..

İmza:

Veli Adı-Soyadı:

Telefon Numarası: